Unstable manifolds for fractional semilinear equations

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Keywords: fractional equations; approximation; unstable manifold.

MSC2010 codes: 34G20, 35A35

This talk is devoted to the numerical analysis of the abstract semilinear fractional Cauchy problem

$$D_t^\alpha u(t) = Au(t) + f(u(t)), \quad u(0) = u^0,$$

in a Banach space E, where $\mathbf{D}_t^\alpha$ is the Caputo-Dzhrbashyan derivative $(0 < \alpha < 1)$, the operator A generates an analytic and compact C_0-semigroup $\exp(\cdot A)$ and the function $f(\cdot)$ is smooth enough [1,2].

We are developing a general approach to establish the existence of unstable manifold for fractional equation and then prove the semidiscrete approximation theorem of unstable manifolds. The phase space in the neighborhood of the hyperbolic equilibrium can be split in such a way that the original initial value problem is reduced to systems of initial value problems in the invariant subspaces [3]. We show that such a decomposition of the equation keeps the same structure under general approximation schemes. The main assumption of our results are naturally satisfied, in particular, for operators with compact resolvents and can be verified for finite element as well as finite difference methods.

Acknowledgments. The work has been supported partially by grant from Russian Science Foundation N20-11-20085.

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