An overview of hyperbolic systems on networks - transport, telegraph and more. Do they differ?

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In the talk we will discuss various manifestations of the initial-boundary value problem of the form

$$\begin{array}{ll} \partial_t \begin{pmatrix} \boldsymbol{\upsilon} \\ \boldsymbol{\varpi} \end{pmatrix}(x,t) = \begin{pmatrix} -\mathcal{C}_+ & 0 \\ 0 & \mathcal{C}_- \end{pmatrix} \partial_x \begin{pmatrix} \boldsymbol{\upsilon} \\ \boldsymbol{\varpi} \end{pmatrix}(x,t), & 0<x<1, t>0, \\ \boldsymbol{\upsilon}(x,0) = \mathring{\boldsymbol{\upsilon}}(x), \boldsymbol{\varpi}(x,0) = \mathring{\boldsymbol{\varpi}}(x), & 0<x<1, \\ \Xi_{out} \begin{pmatrix} \boldsymbol{\upsilon}(0,t) \\ \boldsymbol{\varpi}(1,t) \end{pmatrix} + \Xi_{in} \begin{pmatrix} \boldsymbol{\upsilon}(1,t) \\ \boldsymbol{\varpi}(0,t) \end{pmatrix} = 0, & t>0, \end{array} \tag{1}$$

from the dynamics on a network point of view. Such systems arise e.g. when studying telegrapher's-type equations on a network with general local linear Kirchhoff's boundary conditions (see [1]). It is clear that transport problems on networks, investigated in e.g. [2] and [3], fit into the framework of (1). On the other hand, (1) can be transformed into a pure transport problem using the change of variables $x \mapsto 1-x$ for the function $\boldsymbol{\varpi}$, which can be interpreted as reversing the parametrisation of the edges of the underlying graph. We will see the similarities and differences depending on the type of equation posed on the edges and see how too much abstraction can sometimes be misleading.

References

[1] J. Banasiak, A. Błoch, *Telegraph systems on networks and port-Hamiltonians. I. Boundary conditions and well-posedness*, Evol. Eq. Control Th., doi:10.3934/eect.2021046, 2021.

[2] J. Banasiak, P. Namayanja, *Asymptotic behaviour of flows on reducible networks*, Netw. Heterog. Media, 9 (2014), 197-216.

[3] M. Kramar, E. Sikolya, *Spectral properties and asymptotic periodicity of flows in networks*, Math. Z. 249 (2005), 139-162.

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