Explicit solutions to fragmentation equations with growth or decay

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Introduction. Fragmentation equation describes the process of splitting of large clusters into smaller ones. In many cases, however, the cluster can grow or decay on their own due to e.g., surface sedimentation or erosion, or internal birth or death of the components. The resulting equation governing the distribution $u(x,t)$ of the clusters of size x at time t is given by

$$\begin{aligned} u_t(x,t) \pm (r(x)u(x,t))_x &= -a(x)u(x,t) + \int_x^\infty a(y)b(x,y)u(y,t)dy, \qquad x,t \in \mathbb{R}_+, \\ u(x,0) &= u_0(x), \qquad x \in \mathbb{R}_+, \end{aligned}$$

where a is the fragmentation rate, b describes the distribution of sizes of splitting particles and $\pm r$ is the growth/decay rate, see [1–6]. A comprehensive theory of such equations is given in [7,8]. There is, however, an interest in finding explicit solutions. For pure fragmentation equations a general scheme for finding solutions has been outlined in [1,3,4], see also [8] and a special case of the decay fragmentation equations was dealt with in [5,6]. Here, we extend the idea of [5,6] and provide a unified way of dealing with both cases via functional calculus and operator differential equations. The talk is based on [9].

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