Approximate numbers of the operator embedding potentials in the space of bounded and uniformly continuous functions

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The purpose of this article is to study the behavior of the approximate number operator of the embedding of potentials in the space of bounded and uniformly continuous functions.The potential space $H_E^G \equiv H_E^G(\mathbb{R}^n)$ *is defined as the set of convolutions of potential kernels with functions from the base space*

$$H_E^G(\mathbb{R}^n) = \{u = G * f : \ f \in E(\mathbb{R}^n)\},$$
$$\|u\|_{H_E^G} = \inf\{\|f\|_E : \ f \in E(\mathbb{R}^n), \ G * f = u\}.$$

where E *— a rearrangement invariant space, and the kernel* G *— special kind,*

$$G(x) = G_R^0(x) + G_R^1(x); \quad G_R^0(x) = G_(x)\chi_{B_R}(x); \quad G_R^1(x) = G_(x)\chi_{B_R}^c(x),$$

$$c_1\Phi(r) \leqslant= G(x) \leqslant c_2\Phi(r) \quad r = |x| \in (0, R),$$

$0 < \Phi \downarrow$ in $\mathbb{R}_+$; $\int\limits_0^R \Phi(\rho)\rho^{n-1}\,\mathrm{d}\rho < \infty$, $G_R^1 \in L_1(\mathbb{R}^n) \cap E'(\mathbb{R}^n)$; $E'(\mathbb{R}^n)$ — associated space for $E(\mathbb{R}^n)$. *An estimate is obtained for the approximate numbers of the operator embedding potentials in the space* $C(\mathbb{R}^n)$*the space of bounded and uniformly continuous functions with the norm*

$$\omega_c^k(u;\tau) = \sup\left\{\|\Delta_h^k u\|_c : \ |h| \leqslant \tau\right\}, \quad \tau \in \mathbb{R}_+.$$

Here $\Delta_h^k u(x)$ is the k-th difference with the step $h \in \mathbb{R}^n$ at the point $x \in \mathbb{R}^n$.*Let* A_1 *and* A_2 *be two complex (quasi) Banach spaces and let* $T \in L(A_1, A_2)$ *is a linear and continuous operator from* A_1 *to* A_2 *. Approximate numbers of the operator* T *are defined by the expression:*

$$a_m(T) = inf\{\|T - S\| : \ S \in L(A_1, A_2), rankS \leqslant m\}$$

References

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