On moduli of surface non-singular flows

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Two flows $f^t, f'^t\colon M \to M$ are called *topologically equivalent* if there exists a homeomorphism $h\colon M \to M$ sending trajectories of f^t into trajectories of f'^t preserving orientations of the trajectories. In difference with it, two flows are called *topologically conjugate* if $h \circ f^t = f'^t \circ h$, it means that h sends trajectories into trajectories preserving not only directions but in addition the time of moving. To find an invariant showing the class of topological equivalence or topological conjugacy of each flow from some class of flows means to construct *topological classification* for the class. Note that for some classes their classifications in sense of equivalence and conjugacy coincide; for other classes these classifications completely differ. The second case is about the class that we consider in this paper.

The *Morse-Smale flows* were firstly introduced on the plane in the classical paper of A.A. Andronov and L.S. Pontryagin in [1]. The non-wandering set of such flows consists of a finite number of hyperbolic fixed points and finite number of hyperbolic limit cycles, besides, saddle separatrices cross-sect only transversally (which means that saddle points of a flow on the plain can not be connected by a separatrix). This important class of flows was topologically classified up to equivalence for many times on different manifolds. The most important combinatorial invariants are the *Leontovich-Maier's scheme* [2], [3] for flows on the plane, the *Peixoto's directed graph* [4] for Morse-Smale flows on any closed surface and the *Oshemkov-Sharko's molecule* [5] for Morse-Smale flows on any closed surface.

The next step is to classify up to conjugacy. In [6] it was proved that classes of topological equivalence and topological conjugacy on surfaces coincide for gradient-like flows (i.e. Morse-Smale flows without limit cycles). But any limit cycle generates infinite many conjugacy classes for each equivalence class (even two cycles with different periods cannot be conjugate). For two saddles connected by a separatrix the invariant (the so-called *modulus of stability* or *modulus of topological conjugacy*) was found by J. Palis in [7]. So, the period of a limit cycle is a modulus.

In this talk there is considered the class of non-singular flows on the annulus with only two limit cycles on the annulus's boundary components. For these flows there is proved that they have infinite number of moduli, and the flows of the class are classified up to topological conjugacy in the work.

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