The Incoherent Drying Problem Of An Elastic Body With A Variable Young's Modulus

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Annotation. The problem of drying of a linearly elastic porous body is considered. According to two experimental facts: the first is reducing the size of the body with a decrease in humidity, a phenomenological theory of the dependence of mechanical stresses on relative humidity is constructed; second is with decreasing humidity, Young's modulus increases, is assumed the functional dependence of this module on relative humidity. The initial boundary value problem is formulated, in which , due to the low speed of the drying process , inertial terms are neglected. A model two-dimensional numerical example of the drying of a rectangular area is considered, the numerical solution of which is carried out by the finite element method. The calculation results correspond qualitatively to the experiment with drying of a wetted poroelastic body.

Introduction. The problem of drying solids is described by the diffusion equation (2), therefore the unknown the can serve the relative humidity φ. At the same time many authors [1,2] are consider the coefficients k, c, ρ, which characterize the speed of moisture movement as constant or dependent on coordinates, but they may also depend on the humidity itself φ, what makes the problem nonlinear.

Another factor affecting the drying process is the temperature of the body itself and the environment, since it is known, that the surface is cooled during evaporation. And finally, when determining the coefficients of moisture and heat transfer with the environment, it is necessary to take into account air movement around the area in question or solve the problem not only for the area under study, but also for the aerodynamics of the surrounding air environment. Another aspect of the drying process is the change in the shape and size of the solid.

The formulation of such a problem can be carried out within the framework of an elastic body with the addition of stress dependence or deformations from relative humidity φ, for example, linear. In addition, the elastic properties of the body also change with changes in humidity, this can be taken into account, for example, by using the dependence of the Young's modulus E on relative humidity φ [1,2].

In this paper, an incoherent model of drying of an elastic body is constructed, in which the drying problem is solved on the basis of a non-stationary diffusion equation for relative humidity and problem of elasticity theory, in which, under the assumption of a slow drying process, the inertial terms are neglected, but a smooth functional dependence of Young's modulus on relative humidity is given. The numerical solution for a rectangular area is constructed by the finite element method.

Mathematical formulation of the problem For the components $\mathbf{u}$ of an unknown displacement vector and relative humidity φ, the system of differential equations has the form

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} = 0, \quad \frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} = 0, \quad \frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} = 0, \qquad (1)$$

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$$\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}+\frac{\partial^2\varphi}{\partial z^2}=\frac{c\rho}{k}\frac{\partial\varphi}{\partial t}. \qquad (2)$$

In the reduced system (1) — equilibrium equations without taking into account mass forces, (2) — diffusion equation in which c — coefficient specific heat of the body, ρ — density, k — diffusion coefficient and in this formulation of the problem these parameters are considered constant.

Cauchy's relations

$$\varepsilon_{xx}=\frac{\partial u_x}{\partial x},\quad \varepsilon_{yy}=\frac{\partial u_y}{\partial y},\quad \varepsilon_{zz}=\frac{\partial u_z}{\partial z}.$$

$$\varepsilon_{xy}=\frac{1}{2}\left(\frac{\partial u_x}{\partial y}+\frac{\partial u_y}{\partial x}\right),\quad \varepsilon_{xy}=\frac{1}{2}\left(\frac{\partial u_x}{\partial y}+\frac{\partial u_y}{\partial x}\right),\quad \varepsilon_{xz}=\frac{1}{2}\left(\frac{\partial u_x}{\partial z}+\frac{\partial u_z}{\partial x}\right) \qquad (3)$$

Defining equations taking into account the linear dependence on φ

$$\sigma_{xx}=\lambda\Theta+2\mu\varepsilon_{xx}+\varkappa(1-\varphi),\quad \sigma_{yy}=\lambda\Theta+2\mu\varepsilon_{yy}+\varkappa(1-\varphi),\quad \sigma_{zz}=\lambda\Theta+2\mu\varepsilon_{zz}+\varkappa(1-\varphi),$$

$$\sigma_{xy}=2\mu\varepsilon_{xy},\quad \sigma_{xz}=2\mu\varepsilon_{xz},\quad \sigma_{yz}=2\mu\varepsilon_{yz}, \qquad (4)$$

$$\mu=\frac{E}{2(1+\nu)},\quad \lambda=\frac{E\nu}{(1-2\nu)(1+\nu)},\quad \Theta=\varepsilon_{xx}+\varepsilon_{yy}+\varepsilon_{zz}.$$

E, ν — Young's modulus and Poisson's ratio. $\varkappa$ — coefficient of humidity expansion.

For the Young's modulus E, the following three-parameter dependence on relative humidity is selected φ:

$$E=\gamma\left(\beta\frac{e^{-\alpha(\varphi-0.5)}}{1+e^{-\alpha(\varphi-0.5)}}+1-\beta\right), \qquad (5)$$

where α, β, γ are — parameters.

Mechanical boundary conditions at the boundaries — absence of displacements and absence of stresses, respectively:

$$\mathbf{u}|_{L_1}=0,\qquad \mathbf{n}\cdot\sigma|_{L_2}=0, \qquad (6)$$

Boundary conditions for relative humidity at the borders — constant humidity and drying conditions across the border are maintained:

$$\varphi|_{L_3}=\varphi_0,\qquad \left.\frac{\partial\varphi}{\partial\mathbf{n}}\right|_{L_4}=-h(\varphi-\varphi_1). \qquad (7)$$

If in the second condition in (7) the right part is zero, then the boundary is impenetrable.

Numerical experiment. As an example, we consider a two-dimensional problem for a rectangular area for a model material. Figures 1 and 2 shows the calculation schemes of the problem.

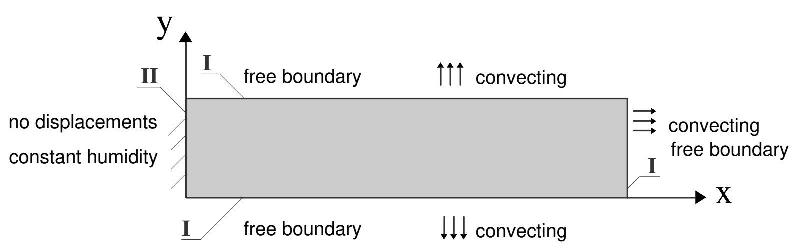


Figure 1: Scheme a.

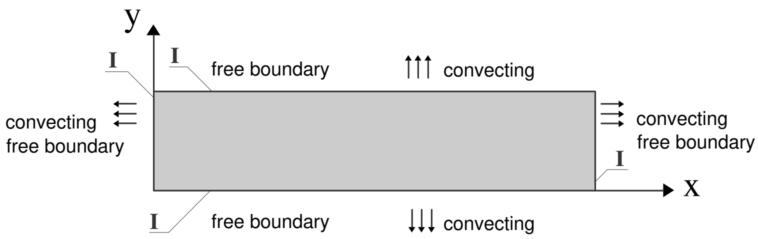


Figure 2: Scheme b.

Numerical simulation was carried out for the following values of physical constants and parameters from (4) and (5)

$$\kappa = 1, \quad k = 1, \quad c = 1, \quad \rho = 1, \quad \alpha = 6, \quad \beta = 0.8, \quad \gamma = 10. \tag{8}$$

and, accordingly, the dependence of Young 's modulus on relative humidity

$$E = 10\left(0.8\frac{e^{-6(\varphi-0.5)}}{1+e^{-6(\varphi-0.5)}} + 1 - 0.8\right), \tag{9}$$

is shown in Figure 3.

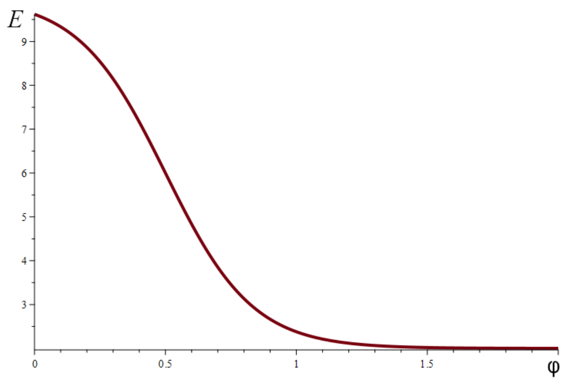
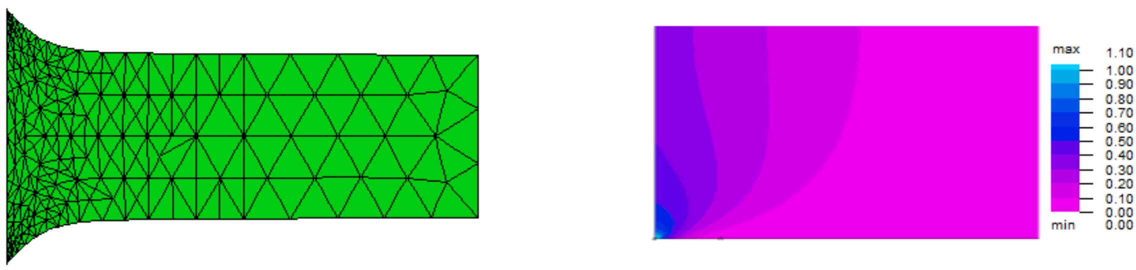


Figure 3: Dependence of Young's modulus on relative humidity.

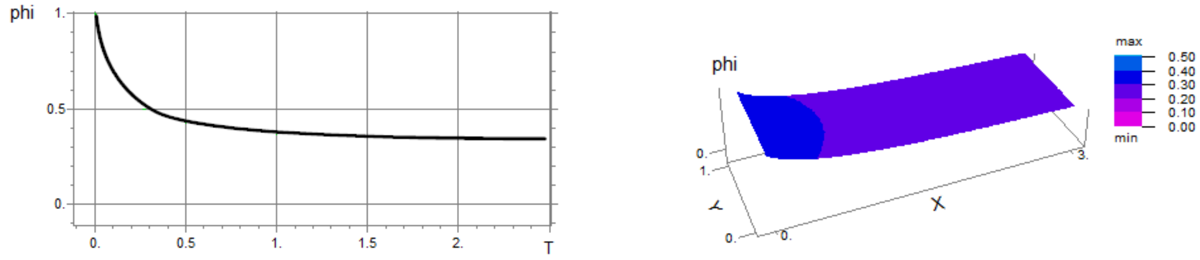
The problem is solved by the finite element method in the ANSYS package, so Figure 4a shows a grid of triangular finite elements with cubic approximation on the deformed state of the domain for a non-stationary problem at the time when the solution goes into stationary mode. Figure 4b shows the Mises stress distribution in the sealing area (Scheme a) at the end of the drying process.



a) Grid on deformed body. b) Scheme b.

Figure 4: Deformed state of the area.

Figure 5a shows the dependence of relative humidity in the center of the region on time, and Figure 5b shows its distribution in the region at the end of the process (Scheme a).



a) $\varphi = \varphi(T)$. Scheme a. b) Distribution of φ. Scheme a.

Figure 5: Relative humidity.

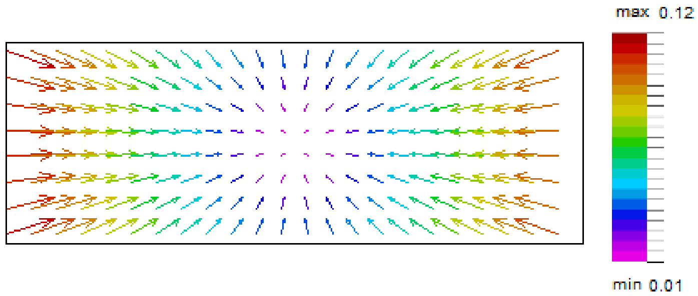


Figure 6: Displacement field.

In the following numerical experiment, the entire boundary of the region is stress-free and the same drying conditions are set on it (Scheme b). At the same time, the shape of the region remains rectangular when entering the stationary mode, Figure 6 shows the vector field of displacements, and the humidity becomes constant in all areas and is equal to the humidity of the environment.

Conclusion. Nonstationary model of drying of a solid deformable body relative to the components of the displacement vector and relative humidity is constructed in this work assuming a smooth dependence of Young's modulus on relative humidity, with the remaining physical coefficients constant.

Numerical experiments were carried out for a rectangular area and a model material to determine the stress-strain state and relative humidity under various boundary conditions. The results of calculations qualitatively coincide with possible field experiments, which indicates the adequacy of the model.

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