Smoothing properties and null-controllability for quadratic evolution equations through the polar decomposition

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In this talk, we will focus on the evolution equations associated with nonselfadjoint quadratic differential operators. On the one hand, it will be explained how the non-commutation phenomena between the selfadjoint and skew-selfadjoint parts of these operators allow the evolution equations studied to enjoy smoothing and localization properties in some specific directions of the phase space, that will be precisely described. On the other hand, it will be found that the properties of null-controllability of these equations are related to a notion of integral thickness condition associated to the skew-adjoint part of the operators involved. These different properties will be deduced from a fine description of the polar decomposition of the evolution operators associated to the studied equations. An application to the generalized Ornstein-Uhlenbeck equations, of which the Kolmogorov and Kramers-Fokker-Planck equations with quadratic external potential are special cases, will be given. These are joint works with J. Bernier (LMJL) and J. Martin (IRMAR).

References

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