Sparsity structures for Koopman operators

C. Schlosser[1] joint work with M. Korda[2]

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Introduction. We consider dynamical systems with a certain sparse structure which states that the system allows (smaller) subsystems (also called factor systems). For a dynamical system on a space X with flow $(\varphi_t)_{t\in[0,\infty)}$ we call a triple (Y, P, ϕ) consisting of a set Y, a surjective map $P : X \to Y$ and a semiflow $(\phi_t)_{t\in[0,\infty)}$ a factor system if for all $t \in [0, \infty)$

$$\phi_t \circ P = P \circ \varphi_t. \tag{1}$$

The functoral nature of the Koopman and Perron-Frobenius operator translate (1) to an intertwining relation for the Koopman and Perron-Frobenius operator. A well-known direct consequence is that eigenfunctions for factor systems induce eigenfunctions of the whole system and invariant measures for the whole system induce invariant measures for the factor systems. The goal of this talk is to give a (partial) answer to the reverse result. We therefore restrict to the situation where $X = \mathbb{R}^n$ and the maps P are canonical projections. With a view towards applications to data science we illustrate the exploitation of such sparse structure at the example of dynamic mode decomposition.

[1]CNRS; LAAS; 7 avenue du colonel Roche, France, F-31400 Toulouse. Email: cschlosser@laas.fr

[2]CNRS; LAAS; 7 avenue du colonel Roche, France, F-31400 Toulouse. Faculty of Electrical Engineering, Czech Technical University in Prague, Technická 2, Czech Republic, CZ-16626 Prague. Email: korda@laas.fr