Subordination principle and Feynman-Kac formulae for generalized time-fractional evolution equations

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Introduction. We consider generalized time-fractional evolution equations of the form

$$u(t) = u_0 + \int_0^t k(t,s)Lu(s)ds$$

with a fairly general memory kernel k and an operator L being the generator of a strongly continuous semigroup. In particular, L may be the generator L_0 of a Markov process ξ on some state space Q, or $L := L_0 + b\nabla + V$ for a suitable potential V and drift b, or L generating subordinate semigroups or Schrödinger type groups. This class of evolution equations includes in particular time- and space- fractional heat and Schrödinger type equations.

We show that the subordination principle holds for such evolution equations and obtain Feynman-Kac formulae for solutions of these equations with the use of different stochastic processes, such as subordinate Markov processes and randomely scaled Gaussian processes. In particular, we obtain some Feynman-Kac formulae with generalized grey Brownian motion and other related self-similar processes with stationary increments.

The talk is based on the joint work with Ch. Bender and M. Bormann.

References:

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