Template for open problems (OPSO 2021 and 2022)

Jochen Glück, Rainer Nagel, Ivan Remizov (eds.)

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Please submit your open problem(s) to Jochen Glück:
jochen.glueck@alumni.uni-ulm.de

Problem 1 (Invariant subspace problem)**.** Let H be a separable Hilbert space over $\mathbb{C}$ and let $T : H \to H$ be a bounded linear operator.

Is there always a closed subspace $\{0\} \subsetneq V \subsetneq H$ which is invariant under T (in the sense that $TV \subseteq V$)?

Comments. If the communicator of the problem is not the same person who originally posed the problem, please mention the original author of the problem here.

Comments on the history and context of the problem. Connections to other problems and/or fields, comments on potential applications and further remarks which explain the relevance of the problem.

For instance: If H is replaced with a general Banach space, the answer to the problem is negative; this was shown by Enflo in 1976 [1].

References:

[1] Per Enflo. On the invariant subspace problem in Banach spaces. *Sémin. Maurey-Schwartz 1975-1976, Espaces Lp, Appl. radonif., Géom. Espaces de Banach, Exposé XIV-XV*, 6 p., 1976.

Communicated by John Doe.

Problem 2. This is an example of a problem without a title. If you would like to include a title, please include it in square brackets right at the beginning of the problem environment (as in Problem 1).

Setting, notation, and other things that are required to understand the problem. Please feel free to insert as much background as necessary (within

the problem environment). Of course, you may also use references within the problem statement, for instance like this [1].

Statement of the problem.

Comments. If the communicator of the problem is not the same person who originally posed the problem, mentioned the original author of the problem here.

Comments about the history of the problem, about related results, about the relevance and context of the problem, about related results and partial results that are already known, and so on.

References to the literature, such as [1] and [1, Theorem 2.1] can also be used here.

References:

[1] Author 1 and Author 2. On very interesting problems. *Journal of examples and counterexamples*, 145(5), 144–156, 1998.

[2] Author. On even more interesting problems. *Rejecta Mathematica*, 144(4), 133–157, 2001.

Communicated by John Doe.