On projections of two-mode networks

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In a *two-mode* (affiliation or bipartite) network $N = ((U, V), L, w)$ the set of nodes is split into two disjoint sets (*modes*) U and V. Each link $e \in L$ has one end-node in the set U and the other end-node in the set V. The function $w : L \to R$ assigns to each link its weight. The network is called *binary* iff $w(e) = 1$ for every $e \in L$. Examples of two-mode networks are attendance of persons to events, membership of persons in institutions, papers linked to their authors, votings of representatives on bills, purchasing of goods by customers, etc. The network N can be described by the corresponding matrix $\mathbf{UV}$. $UV[u, v] = w(u, v)$ for $(u, v) \in L$, and $UV[u, v] = 0$ otherwise.

An approach to the analysis of a two-mode network is its conversion or *projection* to an ordinary (one-mode, weighted) network on a selected set of nodes. This network can be analyzed further using standard network analysis methods.

The standard projection to the second mode V is obtained by multiplying the transposed network matrix with the network matrix, $\mathbf{VV} = \mathbf{UV}^T \cdot \mathbf{UV}$. In a binary network, the entry $VV[v_1, v_2]$ of the projection matrix counts the number of ways we can move in the original network from the node v_1 through some first mode node to the node v_2. There are some problems with the standard projection (Batagelj, 2020). They can be resolved using network normalizations – the *fractional approach*. Especially we point to the role of the first mode nodes of degree 0 or 1.

Another type of projection is based on a (dis)similarity measure d on vectors over R, $VV[v_1, v_2] = d(UV[., v_1], UV[., v_2])$. In many cases, we can show how these measures are related to the standard projection (Batagelj, 2022).

For illustrations, we present the results of applications of projections to some real-life networks.

1. Batagelj, V.: On fractional approach to analysis of linked networks. Scientometrics 123 (2020) 2: 621-633

2. Batagelj, V.: Analysis of the Southern women network using fractional approach. Social Networks 68(2022), 229-236