Local heat kernel: construction and properties

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Heat kernels play crucial roles [1–3] in modern theoretical physics and mathematics, for example, in the Atiyah–Patodi–Singer theorem or in the renormalization of quantum field models. Their explicit construction is possible only in some special cases, so investigation of asymptotic expansions is an important task.

In this talk I am going to discuss a local heat kernel [4] on a smooth Riemannian manifold $\mathcal{M}$, which actually is the main part of the standard heat kernel. In our case the locality means that we work in some smooth open convex domain $U \subset \mathcal{M}$, and the new object does not depend on information from $\mathcal{M} \setminus U$ and any boundary conditions.

This presentation contains a definition of the local heat kernel, its asymptotic expansion, properties of the Seeley–DeWitt coefficients, construction of some useful special functions, and discussion of open questions.

References

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