Improved L^2-approximations in homogenization of parabolic equations with account of correctors

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We consider the Cauchy problem for a second order parabolic equation

$$L_\varepsilon u^\varepsilon = f \text{ in } \mathbb{R}^d \times (0,T), \quad u^\varepsilon(x,0) = 0 \text{ for } x \in \mathbb{R}^d.$$

Here, $L_\varepsilon = \partial_t - div_x\, a(x/\varepsilon)\nabla_x$, the measurable real-valued coefficient matrix $a(x/\varepsilon)$ is ε-periodic and is not necessarily symmetric; ε is a small positive parameter tending to zero; and the right-hand side function f belongs to $L^2(\mathbb{R}^d \times (0,T))$. We find approximations for the solution u^ε in the norm $\|\cdot\|_{L^2(\mathbb{R}^d\times(0,T))}$ with the remainder term of order ε^2. These approximations are of the form

$$u^\varepsilon(x,t) = u(x,t) + \varepsilon U(x, x/\varepsilon, t) + O(\varepsilon^2),$$

where the main term $u(x,t)$ is the solution to the well-known homogenized problem

$$L_0 u = f \text{ in } \mathbb{R}^d \times (0,T), \quad u(x,0) = 0 \text{ for } x \in \mathbb{R}^d,$$

with the parabolic operator $L_0 = \partial_t - div_x\, a^0\nabla_x$ having the constant coefficient matrix a^0 defined via solutions to the auxiliary problems on the periodicity cell which is the unit cube in $\mathbb{R}^d$; the corrector $U(x, x/\varepsilon, t)$, generally, has the three-part structure, that is, $U(x, x/\varepsilon, t) = U_1(x, x/\varepsilon, t) + U_2(x, x/\varepsilon, t) + U_3(x,t)$, and the each part of it is defined with the help of the solutions to the aforementioned cell problems. In the selfadjoint case, the corrector U becomes simpler, because its third term vanishes: $U_3 = 0$.

The above asymptotic for the solution $u^\varepsilon(x,t)$ admits the operator formulation in terms of the resolvent operators L_ε^{-1}, L_0^{-1} and the corresponding correcting operator, which can be restored in accordance with the above corrector $U(x, x/\varepsilon, t)$. Namely,

$$\|L_\varepsilon^{-1}f - L_0^{-1}f - \varepsilon\mathcal{K}_\varepsilon f\|_{L^2(\mathbb{R}^d\times(0,T))} \le C\varepsilon^2 \|f\|_{L^2(\mathbb{R}^d\times(0,T))},$$

where the constant C depends only on the dimension d, the ellipticity constants of the matrix $a(\cdot)$ and the value T.

To obtain these results, we use the so-called shift method proposed firstly in [1,2] and applied to the parabolic homogenization earlier in [3,4].

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