



# A unified approach to degenerate problems in the half-space

L. Negro <sup>1</sup>, G. Metafune <sup>2</sup>, C. Spina <sup>3</sup>

**Keywords:** degenerate elliptic operators, boundary degeneracy, vector-valued harmonic analysis, maximal regularity.

**MSC2020 codes:** 35K67, 35B45, 47D07, 35J70, 35J75

**Abstract.** We study elliptic and parabolic problems governed by the singular elliptic operators

$$\mathcal{L} = y^{\alpha_1} \Delta_x + y^{\alpha_2} \left( D_{yy} + \frac{c}{y} D_y - \frac{b}{y^2} \right), \quad \alpha_1, \alpha_2 \in \mathbb{R}$$

in the half-space  $\mathbb{R}_+^{N+1} = \{(x, y) : x \in \mathbb{R}^N, y > 0\}$ . We prove elliptic and parabolic  $L^p$ -estimates and solvability for the associated problems. In the language of semigroup theory, we prove that  $\mathcal{L}$  generates an analytic semigroup, characterize its domain as a weighted Sobolev space and show that it has maximal regularity.

## References

- [1] G. Metafune, L. Negro, C. Spina. *A unified approach to degenerate problems in the half-space*. Journal of Differential Equations, online preprint arXiv:2201.05573, (2022).
- [2] G. Metafune, L. Negro, C. Spina.  *$L^p$  estimates for the Caffarelli-Silvestre extension operators*. Journal of Differential Equations, Vol. 316, (2022), pages 290-345.
- [3] G. Metafune, L. Negro, C. Spina. *Anisotropic Sobolev spaces with weights*. Tokyo Journal of Mathematics, online preprint arXiv:2112.01791, (2022).
- [4] G. Metafune, L. Negro, C. Spina. *Degenerate operators on the half-line*. Journal of Evolution Equations, 22, 60, (2022).
- [5] G. Metafune, L. Negro, C. Spina. *Schauder estimates for Bessel operators*. Differential and Integral Equations, to appear, (2022).
- [6] G. Metafune, L. Negro, C. Spina. *Elliptic and Parabolic problems for a Bessel-type operator*. Recent Advances in Mathematical Analysis, (2022).
- [7] G. Metafune, L. Negro, C. Spina.  *$L^p$  estimates for a class of degenerate operators*. Discrete and Continuous Dynamical Systems - S, doi: 10.3934/dcdss.2022152, (2022).

---

<sup>1</sup>Dipartimento di Matematica e Fisica “Ennio De Giorgi”, Università del Salento, C.P.193, 73100, Lecce, Italy. Email: luigi.negro@unisalento.it

<sup>2</sup>Dipartimento di Matematica e Fisica “Ennio De Giorgi”, Università del Salento, C.P.193, 73100, Lecce, Italy. Email: giorgio.metafune@unisalento.it

<sup>3</sup>Dipartimento di Matematica e Fisica “Ennio De Giorgi”, Università del Salento, C.P.193, 73100, Lecce, Italy. Email: chiara.spina@unisalento.it