Semigroup methods in regularization of ill-posed stochastic problems

I. V. Melnikova[1]

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Introduction. The conference report is devoted to regularization of ill-posed stochastic Cauchy problems in Hilbert spaces:

$$du(t) = Au(t)dt + BdW(t),\ t \geq 0, \quad u(0) = \xi, \tag{1}$$

where the operator A, in general, is the generator of an R-semigroup in Hilbert space H, in particular, with $-A$ generating a strongly continuous semigroup. The linear operator B acts from the space $\mathbb{H}$, where the process $W = \{W(t), t \geq 0\}$ is defined in the form of series with respect to independent one dimensional Brownian motions, into the space H.

The need for regularization is connected with the fact that the operator A is not supposed to generate a strongly continuous semigroup and with the divergence of the series defining the infinite-dimensional Wiener process W.

We consider regularization of the problem (1) with the operator A, which is the generator of an R-semigroup in a Hilbert space H. The condition for A to be the generator of an R-semigroup in a Banach space means that the solution operators of the corresponding homogeneous Cauchy problem:

$$u'(t) = Au(t),\ t > 0, \quad u(0) = \xi, \tag{2}$$

are generally unbounded in the space and defined only on some subset from the domain $D(A)$, but there is a family of bounded operators called a regularized semigroup or R-semigroup. This family gives a solution to some well-posed problem related to the homogeneous problem, but is not a semigroup in general.

The regularization of ill-posed stochastic Cauchy problems, as in the case of inhomogeneous deterministic problems, is closely related to the regularization of the corresponding homogeneous Cauchy problems.

The conference report consists of four sections.

Section 1 is devoted to the regularization of ill-posed homogeneous problems (2) with sectorial and half-strip operators A such that $-A$ generates a strongly continuous semigroup. In continuation of earlier papers (see, e.g. [1], [2]) two types of regularizing operators $\mathbf{R}_{\alpha,t}$ are considered, that give fundamentally different error estimates of the exact solution to (2) from the solution to a regularized problem with initial data given with an error: $\|\xi - \xi_\delta\| \leq \delta$.

In **section 2** a new approach to constructing regularizing operators $\mathbf{R}_{\alpha,t}$ in terms of R_α-semigroups is introduced. It is shown the connection between regularizing operators $\mathbf{R}_{\alpha,t}$ and R_α-semigroups depending on the regularization parameter α. Nevertheless, the construction of such R_α-semigroups in the general case is not an easy problem.

In **section 3** the Cauchy problem (2) with differential operators $A = A\left(i\frac{\partial}{\partial x}\right)$ is considered. Depending on whether A belongs to different classes in the Gelfand–Shilov classification, R_α-semigroups $\{S_\alpha(t), t \geq 0\}$ with the generator A and matching regularizing operators $\mathbf{R}_{\alpha,t}$ are constructed.

Section 4 is devoted to correctness of infinite-dimensional stochastic problems and regularization of ill-posed stochastic problems.

[1]Ural State University, Institute of Natural Sciences and Mathematics, Russia, Ekaterinburg. Email: Irina.Melnikova@urfu.ru

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