Quantum decoherence via Chernoff averages

R. Sh. Kalmetev [1]

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In this talk we study averages of Feynman-Chernoff iterations [1]

$$e^{-i\hat{H}_n \frac{t}{n}} \circ \ldots \circ e^{-i\hat{H}_1 \frac{t}{n}}$$

for operator-valued quantum evolution functions.

The basic example is provided by Hamiltonians of the form

$$\hat{H}(t) = g(t)\hat{a}^+\hat{a} + f(t)\hat{a}^+ + \overline{f(t)}\hat{a} + h(t), \tag{1}$$

where $g(t)$ and $h(t)$ are real-valued functions of time, $f(t)$ is a complex-valued function of time, $\hat{a}^+$ and $\hat{a}$ are the creation and annihilation operators.

In the works of Glauber [2], Meta and Sudarshan [3] it was shown that in the case of canonical commutation relations the formula (1) defines the general form of Hamiltonians, under which states that are initially coherent remain coherent during the time evolution. By coherent states we mean states corresponding to eigenvectors of the annihilation operator: $\hat{a}z = zz,\ z \in \mathbb{C}$.

Further note that if the Hamiltonian is of the form (1) than the action of the time evolution operator

$$\hat{S}(t) = \mathcal{T}\left\{\exp\left(-i\int_0^t \hat{H}(\tau)\, d\tau\right)\right\}, t > 0,$$

on any initial coherent state is described by the formulas:

$$z(t) = \hat{S}(t)z(0),$$

$$z(t) = e^{-i\phi(t)}z(0) - ie^{-i\phi(t)}\int_0^t f(\tau)e^{i\phi(\tau)}d\tau,$$

where $\phi(t) = \int_0^t g(\tau)d\tau$. The symbol $\mathcal{T}$ denotes the operation of time-ordering.

In some problems [4,5] it becomes necessary to take into account the accumulated common phase factor. Substituting the vector $e^{i\gamma(t)}z(t)$ into the Schrödinger equation leads to the equation

$$\left(i\frac{d\gamma(t)}{dt} + \frac{d}{dt}\right)z(t) = -i\hat{H}(t)z(t),$$

from which it follows that

$$\gamma(t) - \gamma(0) = \int_0^t z(\tau)\hat{H}(\tau)z(\tau) + i\int_0^t z(\tau)\frac{d}{d\tau}z(\tau),$$

where the first term on the right hand side is called a dynamic phase, and the second is called a geometric or Berry phase.

Thus the time evolution of the vectors of the Hilbert space $\mathcal{H}$ corresponding to coherent states under the action of the family of operators $\hat{S}(t)$ is completely described by the operator-valued function with values in the affine group of $\mathbb{R}^3 \cong \mathbb{C} \times \mathbb{R}$.

We consider the time evolution of quantum oscillator, which is given by compositions of random transformations described above, and the diffusion limit of such compositions in the sense of Feynman-Chernoff iterations. We provide the Fokker-Planck equation for the evolution

[1] Keldysh Institute of Applied Mathematics, Russia, Moscow. Email: kalmetev@phystech.edu

of densities of mixed states, and numerically investigate the problem of decoherence of quantum states in interference experiments.

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References

[1] Y.N. Orlov, V.Z. Sakbaev *Feynman-Chernoff Iterations and Their Applications in Quantum Dynamics.*// Proc. Steklov Inst. Math. 2018. Vol. 301. P. 197-206.

[2] R.J. Glauber *Classical behavior of systems of quantum oscillators.* // Physics Letters. 1966. Vol. 21. No. 6. P. 650-652.

[3] C.L. Mehta, E.C.G. Sudarshan *Time evolution of coherent states.* // Physics Letters. 1966. Vol. 22. No. 5. P. 574-576.

[4] R.S. Kalmetev., Y.N. Orlov., V.Z. Sakbaev. *Chernoff Iterations as an Averaging Method for Random Affine Transformations.* // Comput. Math. and Math. Phys. 2022. Vol. 62. P. 996-1006

[5] R.Sh. Kalmetev., Y.N. Orlov., V.Z. Sakbaev. *Generalized Coherent States Representation.* // Lobachevskii Journal of Mathematics 2021. Vol. 42. No. 11 P. 2608-2614