A mild Girsanov formula

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(Work in progress in collaboration with Enrico Priola (Pavia) and Luciano Tubaro (Trento))

ABSTRACT

We consider the SPDE

$$dZ = (AZ + b(Z)dt + dW(t), \quad Z(0) = x$$

on a Hilbert space H, where $A : H \to H$ is self-adjoint, negative and such that A^{-1} is of trace class, $b : H \to H$ is Lipschitz continuous and bounded, and W is a cylindrical process on H. Setting $P_T\varphi(x) = \mathbb{E}[\varphi(Z_x(T))]$ we prove, with the help of formula for nonlinear transformations of infinite dimensional Gaussian measures due to R. Ramer (*J. Functional Analysis*, **15**, 166–187, 1974), the identity

$$P_T\varphi(x) = \int_X \varphi(k(T) + e^{TA}x)\,\rho(x,k)\,N_{\mathbb{Q}_T}(dk), \tag{1}$$

where $N_{\mathbb{Q}_T}$ is the law of W in $L^2(0,T,H)$,

$$\rho(x,k) = \exp\left\{-\tfrac{1}{2}|\mathbb{Q}_T^{-1/2}\gamma_x(k)|_X^2 + M^*(\gamma_x(k))\right\},$$

$$[\gamma_x(k)](t) = \int_0^t e^{(t-s)A}b(k(s) + e^{sA}x)ds$$

and M^* is the adjoint of the Malliavin derivative in X. Finally, letting $T \to \infty$ in (1), we find an explicit formula for the invariant measure ν of P_T, which is ergodic, strongly mixing and absolutely continuous with respect the Gaussian measure $\mu = N_{-1/2\,A^{-1}}$.

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