New Applied Stochastic Models

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In order to study real-life processes or systems it is necessary to choose an appropriate mathematical model for their description, see, e.g. [1], [2]. The usual procedure includes the following steps: 1) Formulate a real problem. 2) Make assumptions. 3) Formulate a mathematical problem. 4) Solve the mathematical problem. 5) Interpret the solution. 6) Validate the model. If one establishes that the model describes correctly the real-life situation the solution can be used to explain, design, predict etc., that is, to make a necessary decision. Otherwise, one has to return to the second step (assumptions) and repeat the procedure once more. This explains, in particular, why there can exist a lot of models describing more or less precisely the same system. The applied probability models have input-output form, that is, they are specified by the following six-tuple $(T, Z, Y, U, \Psi, \mathcal{L})$. Here T is a planning horizon, $Z = \{Z(t), t \in [0,T]\}$ is input process, whereas $Y = \{Y(t), t \in [0,T]\}$ and $U = \{U(t), t \in [0,T]\}$ are output and control processes, respectively. Ψ represents the system configuration and operation mode, hence, $X = \Psi(Z, Y, U)$ is the system state, while $\mathcal{L}_T(U) = \mathcal{L}(Z, Y, U, X, T)$ is objective function (target, valuation criterion, risk measure) evaluating the system performance quality. According to the choice of objective function there exist two main approaches, namely, reliability and cost ones, see, e.g., [3].

The aim of all investigations is to find the optimal control providing extremum (max or min) of the objective function. We are going to explain further procedures in terms of insurance, since it is the oldest applied probability research domain involving risk. So, the most frequently employed controls are reinsurance, investment, bank loans and dividends, see, e.g., [4]-[7]. We study the limit behavior of the insurance company capital under optimal control and carry out the sensitivity analysis of the models under consideration to small fluctuations of parameters and perturbations of underlying processes using the methods proposed in [8], [9].

It is interesting to mention that another interpretation of input and output processes enables us to use the obtained results for other application fields such as communications, inventory, finance, reliability or queueing theory.

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