Environment as a resource for controlling quantum systems

A. N. Pechen [1]

Keywords: quantum control, open quantum systems, incoherent control, engineered environment.

MSC2020 codes: 81Q93, 81S22

Introduction. Control of quantum systems is an important branch of modern quantum physics, whose development is motivated both by fundamental reasons and by existing and prospective applications to quantum technologies [1]. In many experimental circumstances, controlled quantum systems are interacting with the environment [2,3]. The environment is often considered as having deleterious effect on the ability to control the system. However, it can be also exploited as a useful resource. Various approaches for using the environment as a resource exist. In this talk we will discuss the *incoherent control* method proposed in [4]. This method, when combined with coherent control, was shown to provide approximate controllability of generic N–level open quantum systems in the set of all density matrices [5].

Incoherent control by the engineered environment. Density matrix ρ_t at time t of a quantum system under the action of coherent and incoherent controls evolves according to the master equation (we set Planck constant as $\hbar = 1$)

$$\frac{d\rho_t}{dt} = -i[H_0 + H_c(t), \rho_t] + \gamma \mathcal{L}_{n(t)}(\rho_t), \quad \rho_{t=0} = \rho_0 .$$

In this equation, H_0 is the free system Hamiltonian, $H_c(t)$ is the Hamiltonian describing interaction of the system with coherent control $u(t)$ [e.g., a laser field; a typical situation is when $H_c(t) = V u(t)$], $n(t) \geq 0$ is generally time-dependent incoherent control (e.g., spectral density of incoherent photons). The key feature here is that the dissipator $\mathcal{L}_{n(t)}$ becomes dependent on incoherent control. Various physical forms of this dependence were considered in [4]. Equivalently, this master equation was written in [4] as a master equation with time dependent decoherence rates $\gamma_k(t)$,

$$\frac{d\rho_t}{dt} = -i[H_0 + H_c(t), \rho_t] + \sum_k \gamma_k(t) \mathcal{D}_k(\rho_t) .$$

Here k denotes all possible different pairs of energy levels in the controlled system and $\mathcal{D}_k$ is a Gorini–Kossakowski–Sudarshan–Lindblad dissipator, for which two physical classes were exploited — incoherent photons and quantum gas, with two explicit forms of $\mathcal{D}_k$ derived in the weak coupling (describing atom interacting with photons) and low density (describing quantum system interacting with a quantum gas) limits, respectively [4]. Generally, coherent control can also enter in the dissipator, and in opposite, incoherent control also modifies the Hamiltonian via Lamb shift. Non-Markovian master equations can be considered for incoherent control as well.

Applications of incoherent control. The method of incoherent control was found to be successful when applied to various quantum systems. In [5], it was shown that for the explicit form of $\mathcal{D}_k$ derived in the weak coupling limit, generic N-level quantum systems subject to coherent and incoherent controls become approximately controllable in the set of density matrices. The original proposed scheme was significantly speed-up for a two-level case by minimizing time of the incoherent stage [6]. The set of reachable states for a single qubit was described analytically using methods of geometric control theory in [7]. Recently, incoherent

[1]Steklov Mathematical Institute of Russian Academy of Sciences, Department of Mathematical Methods for Quantum Technologies, Russia, Moscow; National University of Science and Technology "MISiS", Russia, Moscow. Email: apechen@gmail.com

control by the environment was combined with speed gradient approach to manipulate energy of a quantum oscillator interacting with the environment [8], where convergence of the differential form of the speed gradient approach to optimal solution was rigorously proved and moreover, the conditions which guarantee that the obtained incoherent control is physical (i.e., non-negative) were found. Various aspects of pure and mixed state preparation using coherent and incoherent controls were investigated also in two-qubit systems [9,10]. All of this show high capabilities of incoherent control [4] for controlling quantum systems.

Acknowledgments. Partial support from the RSF No 22-11-00330, Minobrnauki No 075-15-2020-788, K2-2022-025 and "Priority 2030".

References

[1] C.P. Koch, U. Boscain, T. Calarco, G. Dirr, S. Filipp, S.J. Glaser, R. Kosloff, S. Montangero, T. Schulte-Herbrüggen, D. Sugny, F.K. Wilhelm. *Quantum optimal control in quantum technologies. Strategic report on current status, visions and goals for research in Europe* // EPJ Quantum Technol. 2022. Vol. 9, art. no. 19. https://doi.org/10.1140/epjqt/s40507-022-00138-x

[2] C. Brif, R. Chakrabarti, H. Rabitz. *Control of quantum phenomena: past, present and future* // New J. Phys. 2010. Vol. 12, no. 7. Art. no. 075008. https://doi.org/10.1088/1367-2630/12/7/075008

[3] C.P. Koch. *Controlling open quantum systems: Tools, achievements, and limitations* // J. Phys.: Condens. Matter. 2016. Vol. 28, no. 21. Art. no. 213001. https://doi.org/10.1088/0953-8984/28/21/213001

[4] A. Pechen, H. Rabitz. *Teaching the environment to control quantum systems* // Phys. Rev. A. 2006. Vol. 73, no. 6. Art. no. 062102. https://doi.org/10.1103/PhysRevA.73.062102

[5] A. Pechen. *Engineering arbitrary pure and mixed quantum states* // Phys. Rev. A. 2011. Vol. 84, no. 4. Art. no. 042106. https://doi.org/10.1103/PhysRevA.84.042106

[6] O.V. Morzhin, A.N. Pechen. *On optimization of coherent and incoherent controls for two-level quantum systems* // Izv. RAN. Ser. Mat. 2023. Vol. 87, no. 5. (In press). https://www.mathnet.ru/eng/im9372

[7] L. Lokutsievskiy, A. Pechen. *Reachable sets for two-level open quantum systems driven by coherent and incoherent control* // J. Phys. A: Math. Theor. 2021. Vol. 54, Art. no. 395304. https://doi.org/10.1088/1751-8121/ac19f8

[8] A.N. Pechen, S. Borisenok, A.L. Fradkov. *Energy control in a quantum oscillator using coherent control and engineered environment* // Chaos, Solitons and Fractals. 2022. Vol. 164, Art. no. 112687. https://doi.org/10.1016/j.chaos.2022.112687

[9] V.N. Petruhanov, A.N. Pechen. *Optimal control for state preparation in two-qubit open quantum systems driven by coherent and incoherent controls via GRAPE approach* // Int. J. Mod. Phys. B. 2022. Vol. 37, no. 20–21. Art. no. 2243017. https://doi.org/10.1142/S0217751X22430175

[10] O.V. Morzhin, A.N. Pechen *Optimal state manipulation for a two-qubit system driven by coherent and incoherent controls* (Submitted).