Feynman Integrals in Quantum 2D Gravity

E. T. Shavgulidze [1]

The enormous popularity of 2D gravity in the last several decades motivated by its role in string theory and studies of BH physics in the dimensional reduction approach has grown after realizing the Schwarzian nature of the JT dilaton gravity and the relation of this theory to SYK model.

The general form of the 2D gravity action up to the terms quadratic in curvature K is

$$\tilde{\mathcal{A}} = c_0 \int \sqrt{\mathcal{G}}\, d^2x + c_1 \int K \sqrt{\mathcal{G}}\, d^2x + c_2 \int K^2 \sqrt{\mathcal{G}}\, d^2x \,. \tag{1}$$

The first two terms do not determine the dynamics of 2D gravity. While the part of the action quadratic in the Gaussian curvature does.

Commonly it is transformed to the dilaton gravity action. An alternative way is to deal only with the geometric structures of the surface.

The action (1) is invariant under general coordinate transformations. Here, we reduce the set of coordinate transformations and consider the action restricted to the conformal gauge, where the metric of the 2D surface looks like

$$dl^2 = g(u,\, v)\, \left(du^2 + dv^2\right) = g(z,\, \bar{z})\, dz\, d\bar{z} \qquad \sqrt{\mathcal{G}} = g\,. \tag{2}$$

$$K = -\frac{1}{2g}\, \Delta\, \log g\,, \tag{3}$$

where Δ stands for the Laplacian.

We consider the specific form of the action (1)

$$A = \frac{\lambda^2}{2} \int\limits_d \left(K+4\right)^2\, g(z,\, \bar{z})\, dz\, d\bar{z} = \frac{\lambda^2}{2} \int\limits_d \left(\Delta\psi\right)^2\, dz\, d\bar{z} \tag{4}$$

where

$$\Delta\psi = q\, \Delta \log q + \frac{4}{q}\,, \qquad q = \frac{1}{\sqrt{g}}\,. \tag{5}$$

Now path integrals in the theory

$$\int \tilde{F}(g)\, \exp\{-\tilde{\mathcal{A}}(g)\}\, dg \tag{6}$$

are path integrals

$$\int F(\psi)\, \exp\{-A(\psi)\}\, d\psi \tag{7}$$

over the Gaussian functional measure

$$\mu_\lambda(d\psi) = \frac{\exp\{-A(\psi)\}\, d\psi}{\int \exp\{-A(\psi)\}\, d\psi}\,. \tag{8}$$

We consider a model of 2D gravity with the action quadratic in curvature and represent path integrals as integrals over the $SL(2,\, \mathbb{R})$ invariant Gaussian functional measure. We reduce these path integrals to the products of Wiener path integrals and calculate the correlation function of the metric in the first perturbative order.

Talk is based on V. V. Belokurov and E. T. Shavgulidze, *An approach to quantum 2D gravity, Physics Letters B Volume 836, 10 January 2023, 137633*

[1] Lomonosov Moscow State University, Russia, Moscow.