On optimization of coherent and incoherent controls in one- and two-qubit open systems

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Control of quantum systems, e.g., individual atoms, molecules is an important direction in modern quantum technologies [1–5]. Often open quantum systems with Markovian dynamics are described via the Gorini–Kossakowski–Sudarshan–Lindblad (GKSL) master-equation, and controlling such a system is modelled in terms of coherent control entering in the system's Hamiltonian. However, there is known the approach (see the fundamental works [6, 7] and, e.g., [8–12]), where such a system's environment can be considered as a resource via introducing incoherent control in the superoperator of dissipation and also in the effective Hamiltonian.

The talk considers some one- and two-qubit open quantum systems whose dynamics is described via the GKSL master equation in the weak coupling limit (WCL) approach, and coherent u and incoherent n controls are used:

$$\frac{d\rho(t)}{dt} = -i[H_0 + H_{\mathrm{eff},n(t)} + H_{u(t)}, \rho(t)] + \mathcal{L}_{n(t)}(\rho(t)), \quad \rho(0) = \rho_0, \tag{1}$$

where H_0, $H_{\mathrm{eff},n(t)}$, and $H_{u(t)}$ are, correspondingly, some free, effective, and interaction Hamiltonians; density matrix of the system $\rho(t) \in \mathbb{C}^{N\times N}$ is a Hermitian positive semi-definite matrix, $\rho(t) = \rho^\dagger(t) \geq 0$, with unit trace, $\mathrm{Tr}\rho(t) = 1$; $\mathcal{L}_{n(t)}(\rho(t))$ is the WCL type's superoperator of dissipation acting on $\rho(t)$; $[A, B]$ denote the commutator $[A, B] = AB + BA$ of operators A, B. Consider $N = 2$ and $N = 4$, i.e., correspondingly, for one- and two-qubit cases.

The talk discusses several directions related to some various control problems for the system (1). First, for the problem of generation of a given density matrix ρ_{target} for the one-qubit system, we discuss, based on the article [13], a modification of the two-stage method [7] by using piecewise constant incoherent controls and the two-step gradient projection method at the first (incoherent) stage, where we obtain the possibility to decrease duration of this stage at the cost of complicating the first stage and losing the simplicity of the original method. Second, also for the one-qubit system, we consider such the steering control problem that initial and target density matrices have the same spectrum. We show when we can numerically obtain such a coherent control that, for some initial and target density matrices with the same spectrum, approximately solve the problem and can be used for such an another pair of initial and target density matrices that are related to the first pair due to the certain property. Also we show that increasing the dissipation rate breaks this possibility, and considering both coherent and incoherent controls can help here. This can be considered as a possible modification of the second stage of the two-stage method, and here we consider some special class of incoherent controls for avoiding large variations for each of them. Third, the talk considers the two-qubit system and the problems of minimizing the Hilbert-Schmidt distance between the final and target density matrices, maximizing the Hilbert-Schmidt overlap for them, steering the overlap to a given value [14, 15]. Here we outline the use of the Pontryagin maximum principle, gradient projection methods, stochastic optimization. For the problem of maximizing the overlap, we describe constructing some Krotov's type methods (in terms of density matrices) based on the special exact formulas for the increment of the objective functional [15].

Moreover, the talk notes, as an important direction, the problem of estimating the effectiveness of local and global methods for controlling one- and two-qubit systems.

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