On the uniqueness class and the correctness class of one fourth-order partial differential equation from the theory of heat transfer

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Introduction. In the article [1], in order to describe heat and diffusion processes, a new fourth-order partial differential equation was introduced

$$\alpha_1(\partial_t - \kappa^2\Delta)u(t,\mathbf{x}) + \alpha_2(\partial_t - \kappa^2\Delta)^2 u(t,\mathbf{x}) = f(t,\mathbf{x}), \tag{1}$$

where $\mathbf{x} = (x_1, x_2, \ldots, x_n)$, α_1 and α_2 are some real parameters, $\kappa > 0$ is a physical constant characteristic of the medium, and Δ is the Laplace operator. Also, in the paper [1], the solution of the Cauchy problem for Eq. (1) was formally constructed in the one-dimensional case. However, in the article [1], the most important thing about the Cauchy problem for equation (1) is not presented: the uniqueness class and the correctness class.

Main result. The uniqueness class for the Cauchy problem for Eq. (1) consists of functions g which satisfy the inequality

$$g(\mathbf{x}) \leqslant C\exp(b|\mathbf{x}|^2). \tag{2}$$

The correctness class for the Cauchy problem for Eq. (1) is the is the class of locally integrable functions g, which satisfy the inequality (2).

Thus, Eq. (1) does not improve the uniqueness class and correctness class of the heat equation [3].

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References

[1] V.I. Fushchich, A.S. Galitsyn, A.S. Polubinskii. *A new mathematical model of heat conduction processes.* // Ukr. Math. J. 1990. Vol. 42. No. 2. P. 210–216.

[2] J. Strikwerda. *Finite Difference Schemes and Partial Differential Equations.* Society for Industrial and Applied Mathematics, 2004.

[3] I.M. Gel'fand, G.E. Shilov. *Generalized functions. Vol 3. Theory Of Differential Equations.* Academic Press, 1967.

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