On entropy correct spatial discretizations for 1D regularized systems of equations for gas mixture dynamics

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Introduction. Multicomponent gas mixtures are widespread in nature and industry. Therefore, dynamics of these mixtures is of great theoretical and applied interest. The compressible multicomponent gas mixture dynamics is described by various complicated systems of equations and under different assumptions, for example, in [1]. For considered models, the fulfillment of the entropy balance equation with non-negative entropy production plays the key role in both their physical and mathematical aspects. This property confirms physical correctness of the derived equations and allows one to prove basic a priori estimates of solutions.

Numerical simulation of the gas dynamics is performed by research teams around the world, and a large number of numerical methods was proposed during last decades, for example, in [2]. Methods based on preliminary regularizations of these equations include quasi-gasdynamic (QGD) and quasi-hydrodynamic (QHD) regularizations which are presented, in particular, in [3]. The QGD and QHD equations for the general (multi-velocity and multi-temperature) as well as one-velocity and one temperature gas mixture dynamics were developed and thoroughly studied, and the validity of the entropy balance equations with non-negative entropy production for these systems was proved, see [3-6].

In this report, one-dimensional regularized systems of equations for the general and one-velocity and one-temperature compressible multicomponent inert gas mixture dynamics are considered. Two types of the regularization are studied, and the entropy balance equation is obtained in both cases. The discretization from [7] is generalized, and new nonstandard symmetric three-point spatial discretizations are performed. The suggested discretizations are conservative in mass, momentum, and total energy. Semi-discrete balance equations for the mass, kinetic and internal energies of the mixture are derived as well. The discretizations also satisfy semi-discrete counterparts of the entropy balance equations, and the property of non-negativity of the entropy-production is also proven. The basic discretization in the one-velocity and one-temperature case is constructed by aggregation of the discretization in the case of general mixture, which is a new approach. In addition, an adequate discretization is performed for the terms describing the diffusion fluxes between the mixture components to ensure the non-negative entropy production. The results are obtained together with A.A. Zlotnik and are published in [8].

References

[1] R.Y. Nigmatulin. *Dynamics of Multiphase Media.* Hemisphere. Vol. 1, 1990.

[2] A.G. Kulikovskii, N.V. Pogorelov, A.Y.Semenov. *Mathematical aspects of numerical solution of hyperbolic systems.* Chapman and Hall, 2001.

[3] T.G. Elizarova. *Quasi-gas dynamic equations.* Springer, 2009.

[4] T.G. Elizarova, A.A. Zlotnik, B.N. Chetverushkin. *On quasi-gasdynamic and quasi-hydrodynamic equations for binary mixtures of gases.* // Docl. Math. 2014. Vol. 90. No. 3. P. 719–723.

[5] T.G. Elizarova, A.A. Zlotnik, E.V. Shil'nikov. *Regularized equations for numerical simulation of flows of homogeneous binary mixtures of viscous compressible gases.* // Comp. Math. Math. Phys. 2019. Vol. 59. P. 1832–1847.

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[6] A.A. Zlotnik, A.S. Fedchenko. *On properties of aggregated regularized systems of equations for a homogeneous multicomponent gas mixture.* // Math. Meth. Appl. Sci. 2022. Vol. 45. No. 15. P. 8906-8927.

[7] A.A. Zlotnik. *Spatial discretization of the one-dimensional quasi-gasdynamic system of equations and the entropy balance equation.* // Computational Mathematics and Mathematical Physics. 2012. Vol. 52. P. 1060-1071.

[8] A.A. Zlotnik, A.S. Fedchenko, T.A. Lomonosov. *Entropy Correct Spatial Discretizations for 1D Regularized Systems of Equations for Gas Mixture Dynamics.* // Symmery. 2012. Vol. 14. No. 10. Article 2171.