On convergence rate bounds for linear and nonlinear Markov chains

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A new approach for evaluating convergence rate for general linear and nonlinear Markov chains (MC) will be presented, on the base on the recently developed spectral radius technique for linear MC and on the idea of small nonlinear perturbations for nonlinear MC, [2]–[6].

For *linear MC* this approach uses a so called markovian coupling which provides naturally a certain sub-stochastic matrix or operator, say, V, and the rate of convergence under investigation is determined by its spectral radius $r(V)$. This value is in all cases no greater than the "supremum norm" $\|V\|$, which norm coincides with the so called Markov – Dobrushin ergodic coefficient. The latter coefficient was introduced by Markov for finite matrices in 1906, then it was used by Kolmogorov in 1938, and later by Dobrushin in 1956 in their famous papers; in the literature it is often called Dobrushin's ergodic coefficient, although, in the opinion of the author the name of Markov as the founder of this characteristic is a must. Since the spectral radius is always no greater and often is strictly less than the norm of the operator, the new bound is usually better than the one due to Markov and Kolmogorov: see examples in [5].

To evaluate the rate of convergence for *nonlinear MC,* a new important additional characteristic was proposed in [1]. It was later extended in [3], and eventually a way of using it for nonlinear MC in a combination with the spectral radius approach via small perturbations from linear ones was offered in [6]. The structure of the operator V will be explained in the talk.

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