



Eventual Positivity of Delay Semigroups

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Introduction. In a series of papers see [1, 2, 3] Daniel Daners, Jochen Glück and James B. Kennedy initiated the study of eventually positive C_0 -semigroups of linear operators on Banach lattices, that is, of semigroups for which, for every positive initial value, the solution of the corresponding Cauchy problem becomes positive for large times. They introduced several notions of eventual positivity such as an individual and a uniform one and also gave characterisations of such semigroups by means of spectral and resolvent properties of the corresponding generators. In the paper [4] Daners and Glück studied the eventual positivity of semigroups under bounded perturbations of the generators and concretely demonstrated that the perturbation theory is much more subtle for eventually positive semigroups than it is for positive semigroups. They demonstrated that, in sharp contrast to the case of positive semigroups, eventual positivity of a semigroup is in general lost, if we perturb its generator by a positive operator of large norm. They also showed that individual eventual positivity is not even stable with respect to small positive perturbations.

We study the eventual positivity of semigroups under unbounded perturbations of generators. We prove the eventual positivity of the perturbed semigroups under the unbounded perturbations of generators of analytic, norm continuous and eventually norm continuous semigroups and apply the same to deduce the eventual positivity of Delay semigroups.

References

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