Numerical Solutions for Resolving Groups or Semigroups of Operators for Nonclassical Equations in the Space of Differential Forms

D. E. Shafranov [1], G. A. Sviridyuk [2]

Keywords: differential forms; Riemannian manifold; Laplace – Beltrami operator.

MSC2020 codes: MSC 35R60

Introduction.

Consider the following equations:

– the Barenblatt–Zheltov–Kochina equation [1] $(\lambda - \Delta)u_t = \alpha\Delta u$, which is a model of dynamics of a fluid filtering in a fractured-porous environment;

– the Dzektser equation [2] $(1 - \kappa\Delta)\varphi_t = \alpha\Delta\varphi - \beta\Delta^2\varphi$, which is a model of flow of a viscous-elastic incompressible zero-order Kelvin–Voigt fluid in the first approximation;

– the Ginzburg – Landau equation [3] $(\lambda - \Delta)u_t = \alpha\Delta u + id\Delta^2 u$ from the phenomenological theory of superconductivity.

In the functional spaces $\mathfrak{U}$, $\mathfrak{F}$ chosen by us, this equations are reduced to the linear equation of Sobolev type

$$L\dot{u} = Mu$$

with the irreversible operator L.

Earlier in school on Sobolev type equations the Cauchy problem and the Showalter–Sidorov problem

$$u(0) = u_0, P(u(0) - u_0) = 0$$

for abstract equations were considered.

We propose a transition of linear equation of Sobolev type to the stochastic Sobolev type equations

$$L \overset{\circ}{\eta} = M\eta$$

with the condition

$$\eta(0) = \eta_0, P(\eta(0) - \eta_0) = 0$$

in spaces of Wiener stochastic processes in the case of an abstract (L, p)-bounded operator M, (L, p)-sectorial operator M and (L, p)-radial operator M, respectively. Since Wiener processes are continuous, but non-differentiable in the usual sense at each point, we use the Nelson–Gliklikh derivative. In this article, we study numerical solutions to all three equations (the Barenblatt – Zheltov – Kochina equation, the Dzektser equation and the Ginzburg – Landau equation in spaces of differential forms defined on a torus.

References

[1] D.E. Shafranov *Numerical Solution of the BarenblattвЂ“ZheltovвЂ“Kochina Equation with Additive В«White NoiseВ» in Spaces of Differential Forms on a Torus.* Journal of Computational and Engineering Mathematics. 2019. Vol. 6. No. 4. P. 31вЂ“43.

[2] D.E. Shafranov *Numerical Solution of the Dzektser Equation with В«White NoiseВ» in the Space of Smooth Differential Forms Defined on a Torus.* Journal of Computational and Engineering Mathematics. 2020. Vol. 7. No. 2. P. 58вЂ“65.

[3] D.E. Shafranov *On Numerical Solution in the Space of Differential Forms for one Stochastic Sobolev-type Equation with a Relatively Radial Operators.* Journal of Computational and Engineering Mathematics. 2020. Vol. 7. No. 4. P. 48вЂ“55.

[1] South Ural State University(NRU), Department of Mathematical Physics Equations, Russia, Chelyabinsk. Email: shafranovde@susu.ru

[2] South Ural State University(NRU), Department of Mathematical Physics Equations, Russia, Chelyabinsk. Email: sviridiuga@susu.ru