Higher order traps in quantum control landscapes

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Quantum control, that is control of single quantum systems such as atoms or molecules, attracts now high interest both due to fundamental reasons and applications in modern quantum technologies [1]. Controlled dynamics of an N-level closed quantum system (H_0, V) is described by the Schrödinger equation.

$$i\frac{dU_t^f}{dt} = (H_0 + f(t)V)U_t^f, \qquad U_{t=0}^f = \mathbb{I}. \tag{1}$$

Here H_0 and V ($[H_0, V] \neq 0$) are free and interaction Hamiltonians respectively (i.e., Hermitian $N \times N$-matrices), $f \in L_2([0,T], \mathbb{R})$ is coherent control, and $T > 0$ is some target time. A typical quantum control problem can be formulated as the problem of maximizing the objective functional. In our talk, we consider the following Mayer type quantum control objective functionals:

1. Let O be a quantum observable (system's Hermitian operator) and ρ_0 an initial quantum density matrix (so that $\rho_0 \geq 0$ and $\mathrm{Tr}(\rho_0) = 1$). The objective functional for the expectation of the system observable O is:

$$J_O[f] = \mathrm{Tr}(OU_T^f \rho_0 U_T^{f\dagger}) \to \max . \tag{2}$$

2. The objective functional for generation of a quantum gate $W \in \mathrm{SU}(N)$ is:

$$J_W[f] = \frac{1}{N^2}|\mathrm{Tr}(W^\dagger U_T^f)|^2 \to \max . \tag{3}$$

The goal of global optimization is to find a control which realizes global maximum of the objective. For global optimization, an important question for a controllable system is to establish whether or not the objective has trapping behaviour [2]. *Trap* for an objective functional is a point of local but not global optimum of this functional. The analysis for traps is important since traps, if they exist, determine the level of difficulty for the search for globally optimal controls. If $N = 2$ and $[H_0, V] \neq 0$ then the quantum system (H_0, V) is completely controllable. In this case, the absence of traps was proved in [3,4] for large times. In [5,6], some examples of third order traps were discovered for special N–level degenerate quantum systems with $N \geq 3$. Traps were discovered for some systems with $N \geq 4$ in [7].

In this talk, for the problem of controlled generation of single-qubit phase shift quantum gates we show that control landscape for small times is free of traps [8]. We also discuss the detailed structure of the quantum control landscape for this problem [9]. For the problem of maximizing or minimizing the expectation of a system observable O, we introduce the notion of *trap of n-th order*. We find the conditions under which the control landscape for a strongly degenerate controllable N-level system has trap of the order $2N - 3$ with $N \geq 3$ [10]. It

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is known that this special quantum system is completely controllable [11,12]. Properties of control landscapes for open quantum systems are related to optimization on complex Stiefel manifolds [13].

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