On a class of functionals Feynman integrable in the sense of analytic continuation

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Introduction. This report is devoted to a method for calculating the Feynman integrals of some class of functional. In this case, the integration is carried out over some space E containing both continuous trajectories and trajectories with jumps. The integral over the space E is defined in the sense of analytic continuation. We also obtain a formula is that makes it possible to reduce the calculations of such an integral to calculations of some other integral with respect to the Wiener measure, and this integral is considered on the space of continuous trajectories.

The relationship between the integral over the space E and the integral over the space of continuous trajectories was first found by Belokurov and Shavgulidze [1], but the integral over the space E was considered not in the sense of analytic continuation.

We now recall the definition of the space E introduced in [2]:

Definition 1. $E = \cup_{n=0}^{\infty} X_n$, where X_n is the space of functions $x(t)$ of the form $x(t) = \sum_{j=1}^{n} \frac{1}{t-t_j^*} + \gamma(t)$ where the function γ is Holder on $[0,1]$ with coefficient $\theta \in (0;\frac{1}{2})$ and $\gamma(0) = \sum\limits_{i=1}^{n} \frac{1}{t_i^*}$ and $\gamma(t_k^*) = -\sum\limits_{i\neq k} \frac{1}{t_k^*-t_i^*}$ for $1 \leq k \leq n$.

Definition 2. Consider the functional $f(x) = h(\int\limits_0^1 (\int\limits_0^{t_2} x(t_1)dt_1)\varphi(t_2)dt_2)$ where φ - is a complex-valued continuously differentiable function and h analytic in the whole complex plane function is either of order at most 1 and of a finite type or is of order strictly less than 1. It was proved in [3] that this functional exists on functions of space E. Define $\mathfrak{G}$ as the space of linear combinations of finite products of such functionals.

We now use the definition of the Feynman integral in terms of analytic continuation from the monographs by Smolyanov and Shavgulidze [4].

Theorem 1. Let $f \in \mathfrak{G}$. The functional integral

$$I(\alpha) = \frac{\int\limits_E f(x) e^{-\frac{1}{2}\alpha^2 \int\limits_0^1 (x'(t))^2 dt - \int\limits_0^1 x^4(t)dt + \frac{1}{3}x^3(1)} dx}{\int\limits_E e^{-\frac{1}{2}\alpha^2 \int\limits_0^1 (x'(t))^2 dt - \int_0^1 x^4(t)dt + \frac{1}{3}x^3(1)} dx})$$

has an analytic continuation in the parameter α to the domain

$$\{\alpha | 0 \leq \arg\alpha \leq \frac{\pi}{4}, \frac{1}{2} \leq |\alpha| \leq 2\}).$$

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References

[1] V.V. Belokurov, E.T. Shavgulidze. *Paths with Singularities in Functional Integrals of Quantum Field Theory.* // arXiv:1910.09440 [math.FA] (2013).

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[2] E.S. Kolpakov. *Transformation of the functional integral over discontinuous path to integrals along continuous path.* // Moscow Univ. Math. Bull. J. 2021. . No. 168. P. 76–79.
[3] E.S. Kolpakov. *Reducing the computation of Feynman integrals to integrals in the Wiener measure applying analytic continuation.* // // Tr. MFTI,. J. 2021. . No. 2. P. 130–134.
[4] O.G. Smolyanov, E.T. Shavgulidze. *Continual Integrals.* Nauka, Moscow, 2015.