Superior resilience of non-Gaussian entanglement in local Gaussian semigroup quantum dynamics

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Problem setting and main result. Entanglement distribution task encounters a problem of how the initial entangled state should be prepared in order to remain entangled the longest possible time when subjected to local noises. In the realm of continuous-variable states and local Gaussian channels it is tempting to assume that the optimal initial state with the most robust entanglement is Gaussian too [1,2]; however, this is not the case [3,4]. In Ref. [5] we rigorously prove that specific non-Gaussian two-mode states remain entangled under the effect of deterministic local attenuation or amplification (Gaussian channels with the attenuation factor/power gain κ_i and the noise parameter μ_i for modes $i = 1, 2$) whenever $\kappa_1\mu_2^2 + \kappa_2\mu_1^2 < \frac{1}{4}(\kappa_1+\kappa_2)(1+\kappa_1\kappa_2)$, which is a strictly larger area of parameters as compared to where Gaussian entanglement is able to tolerate noise. These results shift the "Gaussian world" paradigm in quantum information science (within which solutions to optimization problems involving Gaussian channels are supposed to be attained at Gaussian states).

Semigroup dynamics. A considered quantum channel $\Phi(\kappa, \mu)$ with fixed parameters κ and μ represents a snapshot of the dynamical map at a particular time moment t, which may correspond to a finite propagation time through a communication line. In a true time evolution the parameters κ and μ become functions of time t, $\kappa(t)$ and $\mu(t)$. For instance, in the semigroup attenuation or amplification dynamics $\Phi = e^{Lt}$ with the generator L [6] we obtain the following dependencies:

$$\kappa(t) = e^{\pm\Gamma t}, \qquad \mu(t) = \pm\left(e^{\pm\Gamma t} - 1\right)\left(\bar{n} + \frac{1}{2}\right),$$

where the sign $+$ $(-)$ describes amplification (attenuation), $\Gamma \geq 0$ is the process rate, and $\bar{n}$ is the average number of thermal photons in the environment. For such a semigroup dynamics the one-parameter family of maps $\Phi\big(\kappa(t), \mu(t)\big)$ is associated with a straight line in the parameter space (κ, μ).

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