Phase volume invariants of dynamical systems with dissipation

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Tensor invariants (differential forms) for homogeneous dynamical systems on tangent bundles of smooth finite-dimensional manifolds are presented. The connection between the presence of these invariants and the full set of first integrals necessary for the integration of geodesic, potential, and dissipative systems is shown. The force fields introduced into the considered systems make them dissipative with dissipation of different signs and generalize previously considered force fields.

It is well known [1, 2, 3] that a system of differential equations can be completely integrated when it has a sufficient number of not only first integrals (scalar invariants) but also tensor invariants. For example, the order of the considered system can be reduced if there is an invariant form of the phase volume. For conservative systems, this fact is natural. However, for systems having attracting or repelling limit sets, not only some of the first integrals, but also the coefficients of the invariant differential forms involved have to consist of, generally speaking, transcendental (in the sense of complex analysis) functions [4, 5, 6].

For example, the problem of a n-dimensional pendulum on a generalized spherical hinge placed in nonconservative force field leads to a system on the tangent bundle of the $(n-1)$-dimensional sphere with a special metric on it induced by an additional symmetry group. Dynamical systems describing the motion of such a pendulum have the various dissipation, and the complete list of first integrals consists of transcendental functions expressed in terms of a finite combination of elementary functions. There are also problems concerning the motion of a point over n-dimensional surfaces of revolution, the Lobachevsky spaces, etc. The results obtained are especially important in the context of a nonconservative force field present in the system.

In this activity, we present tensor invariants for homogeneous dynamical systems on tangent bundles of smooth finite-dimensional manifolds. The relation between the existence of these invariants and the existence of a complete set of first integrals necessary for the integration of geodesic, potential, and dissipative systems is shown. The force fields introduced into the considered systems make them dissipative with dissipation of different signs and generalize previously considered force fields.

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