Optimization of state transfer and exact dynamics for two-level open quantum systems

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Quantum control which studies methods for manipulation of individual quantum systems is an important tool necessary for development of quantum technologies [1]. Often in experimental circumstances controlled systems can not be isolated from the environment, so that they are open quantum systems. Moreover, in some cases the environment can be used for actively controlling quantum systems, as for example in incoherent control [2,3]. While in some cases the solution for the optimal shape of the control can be obtained analytically, often it is not the case and various numerical optimization methods are needed. A large class of methods are gradient-based numerical optimization algorithms, one of which is GRadient Ascent Pulse Engineering (GRAPE) developed originally for design of NMR pulse sequences [4] and later applied to various problems, e.g. [5,6].

In this talk, we consider the state-to-state transfer control problem for an open two-level quantum system (qubit) whose evolution is governed by the GKSL master equation with coherent and incoherent controls [7,8]. General form of the GKSL master equation in the absence of controls was derived in particular in the weak coupling limit and in the stochastic limit of quantum theory. We consider the specific model of such master equation which includes coherent and incoherent controls. The state of the system is represented by a vector in the Bloch ball. We consider piecewise constant control as it commonly used in gradient optimization methods. Then we derive expressions for dynamics and objective functional gradient using matrix exponentials. Due to low dimension of the system, the corresponding 3×3 matrix exponentials can be analytically diagonalized. For that we find eigenvalues and eigenvectors of the right-hand side matrix of the system evolution equation. Roots of the third order characteristic equation can be analytically found using the Cardano's formula. This enables obtaining exact form of matrix exponentials included in the dynamics and functional gradient expressions necessary for control landscape analysis.

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