

1. Give the definition of hyperbolic node (saddle, sink, source) equilibrium state.
2. Use index theory to prove that equation $z^3 - z = 10$ has at least one complex root.
3. Does there exist a continuous vector field on a surface if the set of singular points of the field consists of exactly l hyperbolic nodes and k hyperbolic saddles and $k - l = -2$? What is the genus of the surface? Provide an example.
4. Does there exist a gradient-like flow on a closed manifold M^3 of dimension $n = 3$, if the set of equilibria of the flow consists of exactly l hyperbolic nodes and k hyperbolic saddles and $k - l = 0$? In case of positive answer, provide an example and specify the topology of manifold.
5. What can you say about invariant manifolds of saddle equilibria of the gradient-like flow on the sphere S^3 , if the set of equilibria of the flow consists of exactly 2 hyperbolic nodes and 4 hyperbolic saddles?
6. What can you say about manifold admitting Morse function, whose set of critical point contains exactly one critical point of each of Morse index $\{0, 1, 2, 3\}$?
7. Construct a gradient-like vector field having two saddle singular points with wild closures of separatrices.
8. Prove that flows, determined by vector fields $\dot{x} = -x$, $\dot{x} = -2x$, $x \in \mathbb{R}^3$, are topologically equivalent.
9. Prove that dynamical systems generated by iterations of maps $f(x) = -2x$; $f'(x) = 3x$, $x \in \mathbb{R}^2$, are non-conjugated.