School-Conference
Mathematical Spring — 2020. Invitation to Dynamical Systems

Book of Abstracts

National Research University
Higher School of Economics

Nizhny Novgorod,
17-21 February 2020

Organizers

National Research University Higher School of Economics, Nizhny Novgorod

Organising Committee

Olga Pochinka

Elena Nozdrinova

Vyacheslav Grines

Contents

Approximate conformal mapping for domain shaped by phase trajectory of the Duffing equation on unit disk

Alekseeva E. S.

Laboratory of infinite-dimensional analysis and mathematical physics

Lomonosov Moscow State University

Let us consider the next Cauchy problem for the Duffing equation:

$$\ddot{x} + x + \varepsilon\, x^3 = 0\,, \qquad x(0) = a_0\,, \qquad \dot{x}(0) = 0\,, \tag{1}$$

under assumption that $0 < \varepsilon \ll 1$ is a small parameter.

In accordance with the theory of perturbations [1] one can find the following approximate solution of equation (1):

$$x(t) = a_0 \cos\omega\, t + \frac{\varepsilon\, a_0^3}{32}\, (\cos 3\,\omega\, t - \cos\omega\, t) + O(\varepsilon^2) \tag{2}$$

where circular frequency ω of nonlinear oscillations is equal to:

$$\omega = 1 + \frac{3\,\varepsilon\, a_0^2}{8} + O(\varepsilon^2)\,. \tag{3}$$

Differentiating formula (2) with respect to time and taking into account expression (3) we obtain the next asymptotic expansion for velocity of motion:

$$\dot{x}(t) = -a_0 \sin\omega\, t - \frac{\varepsilon\, a_0^3}{32}\, (11\, \sin\omega\, t + 3\, \sin 3\,\omega\, t) + O(\varepsilon^2)\,. \tag{4}$$

Formulae (2) and (4) give us approximate parametric representation for phase trajectory of system (1), this phase trajectory being close to circle with radius a_0. Therefore we can construct approximate conformal mapping of the nearly circular domain bounded by this phase trajectory on unit disk on the basis of technique developed in article [2, 3].

To apply this technique one ought to calculate using formulae (2) and (4) the following value:

$$r(\theta) = \sqrt{\frac{\dot{x}^2 + x^2}{a_0^2}} = 1 + \frac{\varepsilon\, a_0^2}{32}\, (5 - 4\, \cos 2\,\theta - \cos 4\,\theta) + O(\varepsilon^2)\,. \tag{5}$$

where $\theta = \omega\, t$.

This value is required for the formula expressing approximate conformal mapping of the nearly circular domain on unit disk obtained in [2, 3] namely:

$$w = \frac{z}{a_0}\left\{ 1 + \frac{1}{2\,\pi} \int_0^{2\,\pi} [1 - r(\theta)]\, \frac{\exp(i\,\theta) + z/a_0}{\exp(i\,\theta) - z/a_0}\, d\theta \right\} + O(\varepsilon^2)\,. \tag{6}$$

Substituting expression (5) into formula (6) one can easily calculate that

$$w = \frac{z}{a_0} + \frac{\varepsilon\, a_0^2}{32}\, \frac{z}{a_0}\left(-5 + 4\,\frac{z^2}{a_0^2} + \frac{z^4}{a_0^4} \right) + O(\varepsilon^2)\,. \tag{7}$$

This work is a part of more general investigation aimed at comparison of usage of effectivisation of the Riemann theorem given in paper [4] applied to the domain bounded by the level curve of exact integral of energy for equation (1):

$$\frac{\dot{x}^2}{2} + \frac{x^2}{2} + \frac{\varepsilon\, x^4}{4} = \frac{a_0^2}{2} + \frac{\varepsilon\, a_0^4}{4}$$

under small positive ε with formula (7).

References

[1] L.D. Landau and E.M. Lifshitz, Mechanics (Volume 1 of A Course of Theoretical Physics), Pergamon Press, 1969.

[2] M.A. Lavrentyev, Sur une méthode géométrique dans la représentation conforme, Atti Gongr. internat. Bologna, 1928, 3, 241.

[3] G.V. Siryk, On a conformal mapping of near domains, Uspekhi Matem. Nauk, 9 (1956), no. 5, pp. 57-60 [in Russian].

[4] S.M. Natanzon, Formal series for the τ-function realizing Riemann's theorem about domains in the complex plane, Russian Math. Surveys, 56:4 (2001), pp. 760–761.

Classification of chaotic attractors in multi-circuit radio-physical generator

Batishchev V. P.[1], **Stankevich N.V.**[1,2]

1. Department of Radio-Engineering and Telecommunications

Yuri Gagarin State Technical University of Saratov

2. Laboratory of Topological Methods in Dynamics

National Research University High School of Economics

The development and study of radio-physical generators of complex oscillations is one of the perspective problems, both from a practical and theoretical point of view. One of the promising areas of practical use of such radio-physical generators is their use in telecommunication systems [1]-[3].

As the simplest autonomous generator, in which it is possible to implement chaotic signal resulting from multi-frequency quasi-periodic oscillations, a multi-circuit generator can be used. In [4], a multi-circuit oscillator model was proposed, in which the possibility of multi-frequency quasiperiodic oscillations, as well as the occurrence of chaotic dynamics, was demonstrated. Such a generator consists of n circuits coupled via a common control circuit. Each oscillatory circuit has its own frequency, as well as a parameter responsible for its excitation, i.e. for exciting every mode in the generator.

Such a generator can be written in the following system of differential equations:

$$\ddot{x}_i - (\lambda k_i - x_i^2)\dot{x}_i + \Delta_i x_i + \sum_{i=1}^{n} k_i \dot{x}_i - k_i \dot{x}_1 = 0, \tag{1}$$

where $i = (1..N)$, the number of circuits in the generator, x_i, $\dot{x}_i$ are the dynamic variables of each oscillatory circuit. Each oscillatory circuit is a van der Pol-type oscillator in which the parameter λ is responsible for the excitation of self-oscillations in the circuit. The parameters Δ_i determine the frequencies of each oscillatory circuit. We considered the case of a five-circuit generator, i.e. $N = 5$ and the parameters responsible for the base frequencies were fixed as irrational, equidistantly distributed: $\Delta_1 = 1$, $\Delta_2 = \sqrt{3}$, $\Delta_3 = \sqrt{11}$, $\Delta_4 = \sqrt{41}$, $\Delta_5 = \sqrt{153}$.

The main tool to analyze complex oscillatory regimes, including chaotic and quasiperiodic, is the analysis of the full spectrum of Lyapunov exponents [5]. An analysis of the spectrum of Lyapunov exponents allows to: distinguish chaotic oscillations; diagnose quasiperiodic oscillations with a different number of incommeasure frequencies; determine the type of quasiperiodic bifurcation (Hopf or saddle-node); classify various types of chaotic dynamics. A preliminary analysis of the five-circuit generator (1) showed that quasiperiodic oscillations with a different number of frequency components from five to one can occur in such a system [4]. Moreover, with an increase in the parameters responsible for the amplification of each mode, only two-frequency quasiperiodic oscillations and chaos are preserved in the system resulting from the destruction of the two-frequency torus with one positive and one zero Lyapunov exponents. At small values of the amplification coefficient, quasiperiodic oscillations with a different number of frequencies are preserved, and when they are destroyed, chaotic attractors with a different spectrums of Lyapunov exponents are formed. In the frame of this work analysis of different chaotic attractors of model (1) will be presented, with phase portraits, Poincaré sections, Fourier spectrums and Lyapunov exponents.

References

[1] Pecora L.M., Carroll T.L. // Phys. Rev. Lett., 1990, 64(8), 821-824.

[2] Dmitriev A.S., Panas A.I. Dinamicheskiy haos: novye nositeli informatsii dlya sistem svyazi. M.: Fizmatlit, 2002, 252 p. (in Russia).

[3] Koronovskii A.A., Moskalenko O.I., Hramov A.E. // Physics-Uspekhi, 2009, 52(12), 1213-1238.

[4] Stankevich N.V., Astakhov O.V., Kuznetsov A.P., Seleznev E.P.// Technical Physics Letters, 2018, 44(5), 428-431.

[5] Pikovsky A., & Politi A. Lyapunov exponents: a tool to explore complex dynamics. Cambridge University Press. 2016.

On Realization of Gradient-Like Flows with Three Equilibria

Chernov A. A.

Department of Fundamental Mathematics

National Research University Higher School of Economics

A smooth flow f^t given on closed smooth manifold M^n is called gradient-like if its non-wandering set Ω_{f^t} consists of finite number of hyperbolic equilibria and an intersection of invariant manifolds of equilibria is transversal. It follows from [1] that for any gradient-like flow f^t the set Ω_{f^t} contains at least one source and one sink equilibria. If the set Ω_{f^t} is exhausted of this two points then the ambient manifold M^n is the sphere and all such flows are topologically equivalent. We consider a class $G_3(M^n)$ of gradient-like flows whose non-wandering set consists of exactly three points.

It follows from Poincare-Hopf Theorem that for $n = 3$ the class $G_3(M^n)$ is empty. The unique 2-dimensional manifold admittint such flows is the projective plane. Due to [2], all flows from $G_3(M^2)$ are topological equivalent.

According to [3, 4], for any gradient-like flow there exists an energy Morse function. Hence the flows from the class $G_3(M^n)$ exists only on manifolds admitting Morse function with exactly three critical points. Topology of such manifolds was studied in [5]. In particular, there was proved that $n \in \{2, 4, 8, 16\}$, critical points of the Morse function has indexes $0, \frac{n}{2}, n$, and M^n is a union of the open n-ball and the sphere of dimension $\frac{n}{2}$.

In [6] necessary and sufficient conditions of topological equivalence of flows from $G_3(M^n)$ were obtained. In particular, it was proved that for $n = 4$ all flows from $G_3(M^n)$ are topologically equivalent and an algorithm of construction of flows from $G_3(M^n)$ was provided. The fact that there is only one class of topological conjugacy in $G_3(M^4)$ leads to uniqueness of the ambient manifold M^4, while for $n \in \{8, 16\}$ there are non-homeomorphic manifolds admitting flows from the considered class.

In the report a new method of realization of flows from $G_3(M^n)$ for $n = 2, 4$ is proposed. We use the idea of constructing manifolds by a sequence of gluing handles. Remain that an n-dimensional handle of index k is the n-ball $H_k^n = B^k \times B^{n-k}$ presented as the direct product of balls B^k, B^{n-k}. The set $F_k^n = \partial B^k \times B^{n-k}$ is called the foot of the handle. Define the flow f_k^t on the handle H_k^n by the system

$$\begin{cases} \dot{x} = x, x \in B^k \\ \dot{y} = -y, y \in B^{n-k}. \end{cases}$$

Then to get the desire manifold M^n carring the the flow $f^t \in G_3(M^n)$ glue handle H_2^n to the hanlde H_0^n by means a diffeomorphism $\varphi : F_k^n \to \partial H_0^n$ on the image to obtain a manifold with boundary diffeomorphic to the sphere S^{n-1}. It allow to glue the handle H_n^n to obtained manifold and to get the desire flow. The key of the construction is the choice of an appropriate diffeomorphism φ.

References

[1] S. Smale, "Differentiable dynamical systems" // Bull. Amer. Math. Soc. 1967. No. 73. P 747–817.

[2] M.M. Peixoto, "On a classification of flows on 2-manifolds" // Proc. Symp. Dyn. Syst. Salvador. 1973. P 389–492.

[3] S. Smale, "On gradient dynamical systems" // Annals Math. 1961. No. 74. P 199–206.

[4] K.R. Meyer, "Energy Functions for Morse Smale Systems" // American Journal of Mathematics. 1968. No. 90:4. P 1031–1040.

[5] J. Eells, N. Kuiper, "Manifolds which are like projective planes" // Institut des Haute studes Scientifiques Publications Mathematiques. 1962 No. 14. P 5–46.

[6] Zhuzhoma E. V., Medvedev V., "Continuous Morse-Smale flows with three equilibrium positions" // Mathematical collection. 2016. Vol. 207. No. 5. P. 69–92.

Mathematical Basics of Content-Aware Scaling

Ivanov V. V.[1], Lebedikova V. E.[2]

1. Department of Computer Systems and Software Engineering
Peter the Great St.Petersburg Polytechnic University

2. Department of Computer Systems and Software Engineering
Peter the Great St.Petersburg Polytechnic University

Image scaling is a major tool in image processing applications. Due to the diversity of electronic devices, compressing and enlarging images remains an important task.

There are two most common methods for image scaling:

1) image scaling, but it doesn't take into account the context and importance of objects and leads to deformations;

2) image cropping, but this method can remove pixels only along the edges of the image;

We explore the lesser-known "seam carving"[1] algorithm that "cutting out seams" for smart changes in the aspect ratio of the image, removing only insignificant content without distortion of important parts.

A seam is a path of pixels from one edge of the image to the other. The idea of algorithm is to cut out the seams that contain the least information for the user. The unit of this information is "energy".

Seam carving algorithm consists of two steps:

1) create energy map by assigning energy to each pixel using some energy function;

2) using the recurrence relation and dynamic programming find the seam with the least energy and delete it from picture;

This study aims to the examine various energy functions from the theoretical side and then compare them in the context of application for seam carving.

The energy functions are better known as edge detection functions in field of computer vision. There are three distinct kinds of edge detection algorithms:

1. First-order image derivative based[2] (Sobel and Prewitt operators, see Fig.1). Based on searching for local directional maxima of the gradient magnitude using the gradient direction. A discrete analogue of the first-order derivative is used, which is approximately computed by convolution with various kernels.

2. Second-order image derivative based[3] (Laplacian of Gaussian). Based on search for zero crossings in a second-order derivative expression computed from the image in order to find edges, usually the zero-crossings of the Laplacian of image. Very susceptible to image noises, so a image smoothing is a crucial pre-processing step (usually Gaussian smoothing is used).

3. Entropy-based[4] (Renyi entropy). Works on a gray image, splitting it into equivalence classes according to distributions of pixels of each shade of gray.

Derivative-based functions use discrete 2-variable version of function convolution with some kernel (unique to each function), as the image can be seen as discrete function of 2 variables, that given the coordinates gives you the colour of appropriate pixel. Convolution allows pixels around the selected one to influence the result.

We reviewed and implemented each type of function in Haskell language (see Fig.1). After testing with different image types (portrait, landscape), we came to the following conclusions: of all functions, the first-order derivative based are most suitable for seam carving. Second-order derivative based are very susceptible to image noise and defects that will accumulate during seam carving. Entropy based functions are unsuitable as energy functions, because they split the image into only two equivalence classes by one shade of gray, which is not enough.

In addition, during the study we improved some details of the reviewed algorithms and got new ideas. Seam carving can be used to expand the image by creating new seams from adjacent pixels using complicated algorithm, that we simplified without drastically reducing quality. As a future study, it is planned to improve the seam carving algorithm by combining it with a image saliency map (a map of "importance" of objects in the image).

References

[1] Avidan S., Andshamir A. Seam carving for content-aware image resizing. ACM Trans. Graph. 26, 3, 10, 2007

[2] Rashmi, Mukesh K., Rohini. Algorithm and Technique on Various Edge Detection : A Survey. An International Journal (SIPIJ) Vol.4, No.3, June 2013

[3] Laplacian of Gaussian [Electronic resource]. URL: https://homepages.inf.ed.ac.uk/rbf/HIPR2/log.htm (Date accessed: 20.11.2019)

[4] Singh B., Singh A. P. 2008. Edge Detection in Gray Level Images based on the Shannon Entropy. Journal of Computer Science 4 (3): 186-191, 2008

Dynamics of diffused connected systems of differential equations with internal connection

Ivanovsky L. I.

Faculty of Computer Science

P.G. Demidov Yaroslavl State University

Let us consider differential equation system with diffusion interaction between adjacent elements and internal connection

$$\dot{u}_j = N^2(u_{j+1} - 2u_j + u_{j-1}) + \gamma u_j, \qquad j = \overline{1, N}, \tag{1}$$

$$u_0 = u_1, \; u_{N+1} = u_N + \frac{\alpha}{N}u_k + \frac{\beta}{N}u_k^3, \quad 1 \leq k < N, \tag{2}$$

where u_j is a smooth function, $t \geq 0$, and parameters α, β, γ are real numbers.

In system (1), (2) there are implemented two cases of stability loss of the homogeneous zero solution $u_j(t) \equiv 0$: divergent, when the zero value appears in the spectrum of stability, and oscillating, when the spectrum of stability has a pair of complex eigenvalues with maximal real parts equal to 0. The task of research was to study the nature of stability loss of zero solution and deduce asymptotic formulas for the regimes which derive from zero solution of system (1), (2) for critical values of parameters α, γ.

The obtained analytical results were illustrated by the numerical solution of system (1), (2) for values of parameters close to bifurcational ones. In system (1), (2), for values of parameter α close to critical ones, the normal form was given. This form allowed to determine conditions of the apperance of inhomogeneous balance states and cycles near the zero solution of system (1), (2).

On Afraimovich-Shilnikov torus-chaos attractors in one economic model

Karatetskaia E., Kazakov A.

1. Laboratory of Dynamical Systems and Applications National Research University Higher School of Economics

The class of New Keynesian models is quite widespread as a toolkit for studying macroeconomic policy. Based on the assumption that economic downturns are caused by processes of market inefficiency, the New Keynesian theory provides a rational basis for state intervention in the economy, such as countercyclical monetary or fiscal policies [1].

Behavioral New Keynesian models devoted to the interaction and mutual influence of monetary policy and the behavior of economic agents allow to study effects not only within the framework of econometric and economic methods, but also using mathematical approaches, including the application of the theory of dynamical systems and the theory of bifurcations. Understanding the dynamic behavior of variables in a model helps to draw some conclusions that are also important in an economic sense. A number of recent works have carefully studied some bifurcations arising in the framework of behavioral New Keynesian models, such as Andronov-Hopf and period-doubling bifurcations: there are some results that show that the parameters of monetary policy, as well as some characteristics of economic agents such as the speed of their learning and the costs of rationality, lead to loss of stability of the model and, in some cases, transition to chaotic dynamics [2,3].

In this work we study one of the simplest New Keynesian models describing by a two-dimensional map. We show that loss of stability of fixed point corresponding to zero growth of inflation in this model appears due to Andronov-Hopf bifurcation. As a result, stable invariant curve (invariant torus) appears in this model. We demonstrate, that with further increase in elasticity of monetary policy and parameters corresponding to the number of fully or bounded rational agents in the economy, this stable curve is destroyed according to Afraimovich-Shilnikov scenario [4]. In the framework of this scenario we show different routes to torus-chaos attractors in this map.

ReferencesReferences

References

[1] Branch, W. A., McGough, B. (2018). Heterogeneous expectations and micro- foundations in macroeconomics. In Handbook of computational economics (Vol. 4, pp. 3-62). Elsevier.

[2] Branch, W. A., McGough, B. (2010). Dynamic predictor selection in a new Keynesian model with heterogeneous expectations. Journal of Economic Dynamics and Control, 34(8), 1492-1508.

[3] Jump, R. C., Hommes, C., Levine, P. (2019). Learning, heterogeneity, and complexity in the New Keynesian model. Journal of Economic Behavior; Organization, 166, 446-470.

[4] Afraimovich, V. S., Shilnikov, L. P. (1983). On invariant two-dimensional tori, their disintegration and stochasticity. Methods of the qualitative theory of differential equations, Gos. Univ. Gorkij, Gor'kov.

On one method of constructing hyperbolic and pseudohyperbolic attractors on a torus

Kazakov A.O., Grines V.Z., Mints D.I.

International Laboratory of Dynamic Systems and Applications
National Research University Higher School of Economics, Nizhny Novgorod, Russia

Surgical operations on two-dimensional and three-dimensional tori leading to the birth of hyperbolic and pseudohyperbolic attractors are a fairly well-studied area of the theory of dynamical systems. Nevertheless, the study of the properties of attractors arising from these surgical operations is of considerable interest. In this report we propose a method for producing bifurcations which are analogous to surgical operations, as a result of which hyperbolic DA-attractor or pseudohyperbolic attractor is generated on the torus. The construction of these attractors is carried out making use of the composition of the linear Anosov diffeomorphism and the Möbius map.

It is a well-known fact that the Anosov map has a fixed saddle point O, whose invariant manifolds intersect transversally and their closure forms a dense set on the torus. The saddle point O is conservative, i.e. the product of its eigenvalues is equal to 1. The Möbius map proposed by A.S. Pikovsky is used as a perturbation. It is worth noting that this map is given by a smooth one-dimensional diffeomorphism on the circle, depending on parameter $\varepsilon \in [-1, 1]$. For $\varepsilon = 0$ the Möbius map is identical, for $\varepsilon \neq 0$ it has stable and unstable fixed points. As a result of applying the Möbius map to one or several coordinates in the Anosov map we obtain a non-conservative diffeomorphism (the product of eigenvalues of the fixed point O is not equal to 1 for $\varepsilon \neq 0$). By varying the perturbation parameter ε, one can obtain bifurcations leading to a change in the type of the fixed point O in the resulting diffeomorphism.

In this report we consider two type of bifurcations: pitch-fork bifurcations and Neimark-Sacker bifurcations.

1. Pitch-fork bifurcations.

   As a result of the pitch-fork bifurcation the fixed point O becomes unstable (source) and a pair of saddle points appears in its neighborhood. An unstable manifold of these points cuts out the Cantor set on the torus and a DA-attractor appears. We study these attractors in case of two-dimensional and three-dimensional tori. We believe that on three-dimensional torus such bifurcation can lead to the birth of an attractor with two directions of hyperbolic instability, i.e. with two positive Lyapunov exponents. Thus, hyperchaotic DA-attractor can be obtained.

2. Neimark-Sacker bifurcations.

   As a result of the Neimark-Sacker bifurcation the saddle (in this case, saddle-focus) fixed point O becomes unstable and saddle invariant curve with two-dimensional stable and two-dimensional unstable manifolds appears in its neighborhood. In the case under consideration, the two-dimensional unstable manifold of the saddle invariant curve cuts out a Cantor set of the Sierpinski carpet type on a three-dimensional torus and a pseudohyperbolic attractor appears. Note that this attractor is not hyperbolic since it contains saddle invariant curves with neutral direction which breaks the continuity of two-dimensional contracting subspace. Also, by varying the parameters, one can obtain various resonant cases of Neimark-Sacker bifurcation. In particular, the case of resonance is presented in the result of which in the neighborhood of the fixed point O 4 saddle points of the type (2,1) (i.e. with two-dimensional stable and one-dimensional unstable invariant manifolds) and 4 saddle points of the type (1,2) are born.

We also study the properties of the obtained attractors making use of numerical methods. In particular, using the analysis of the continuity of the direction-contracting and volume-expanding subspaces, it is shown that the obtained attractors are indeed hyperbolic or pseudohyperbolic.

Shadowing in a linear cocycle over Bernoulli shift Monakov G.

Starting from shadowing lemma by Anosov and Bowen it is well-known that shadowing property is closely related to structural stability. After numerical study of shadowing property in not structurally stable systems Hammel, Yorke and Grebogi formulated following conjecture: a typical dissipative map with nonzero Lyapunov exponents satisfies finite Holder shadowing property [1]. We prove this conjecture for a continuous linear cocycle over Bernoulli shift. The main technique is large deviation principle for mixing systems.

References

[1] Hammel, S., Yorke, J., Grebogi., C., (1987). Do numerical orbits of chaotic dynamical processes represent true orbits. вЂ“ J. of Complexity 3, 136-145.

Research of random and pseudo random number generation methods

Neverov V. S.[1], Kiun A. A.[2]

1. Department of Computer Systems and Software Engineering
Peter the Great St.Petersburg Polytechnic University

2. Department of Computer Systems and Software Engineering
Peter the Great St.Petersburg Polytechnic University

The generation of large arrays of random numbers is critical for the safe transfer of data and science, as well as games and casinos [1].

There are two main approaches to generating random numbers:

1) using a device that generates a sequence of random numbers based on randomly varying parameters of physical processes (random number generation, hereinafter referred to as the RNG);

2) using mathematical algorithms that produce a pseudo-random sequence (generation of pseudo-random numbers, hereinafter referred to as the PRNG).

The difference is that as any pseudo-random number is obtained by calculations according to some algorithm, it is possible to predict which number will be generated for pseudo-random number generators.

This study aims to create a new random number generation algorithm and compare it with existing ones. Tests provided by the National Institute of Standards and Technology (NIST) (namely, an environment capable of performing fifteen statistical tests that evaluate the probability that a perfect source of entropy will produce a sequence that is less random than the sequence being tested [2]) were used for checking the effectiveness of the designed algorithm.

Our method of generating random numbers is implemented in two consecutive stages.

At the first stage, a sequence of truly random numbers is generated based on user actions with a script in the Javascript language. The script can be embedded in absolutely any website, where it generates a sequence of random numbers using the last two digits of the number of milliseconds passed since the visitor entered the site and sends it to the server for processing and writing into a file. The sequence in the resulting file consists of random numbers of random length from random users of random sites.

The productivity of such a system is potentially unlimited, since at the moment the Internet is used by approximately 3.9 billion people around the world. However, in order to make the method even more productive, the second stage was developed.

At the second stage, 10 numbers are taken from the sequence obtained at the first stage and are used as a seed for mathematical algorithm Alea (PRNG). Based on the seed, 25 thirteen-digit numbers are generated. This allows to increase the original sequence 32.5 times, while maintaining the required level of random properties, although the resulting sequence is not entirely random.

During the experimental verification, the script was installed on three sites where people were invited.

As a result, it was revealed that the algorithm we developed passes all the NIST tests, in addition, the distribution of the digits of the generated sequence is very even.

We also compared the developed algorithm with the Mersenne twister (PRNG, which provides fast generation of high-quality pseudorandom numbers [3]). As a result, it was found that our algorithm generates better numbers by the criterion of randomness.

Thus, as a result of the study a method of generating a large array of random numbers based on user actions was developed and tested in real conditions.

References

[1] Applications of randomness. [Electronic resource]. URL: https://en.wikipedia.org/wiki/Applications_of_rando (Date accessed: 11.17.2019).

[2] NIST SP 800-22 A statistical test suit for Random and Pseudorandom Number Generators for Cryptographic Applications URL: https://nvlpubs.nist.gov/nistpubs/Legacy/SP/nistspecialpublication800-22r1a.pdf (Date accessed: 17.11.2019)

[3] Mersenne Twister [Electronic resource]. URL: https://en.wikipedia.org/wiki/Mersenne_Twister(Date accessed: 16.11.2019

Predicting time intervals in an online course system based on machine learning methods Nigmatulin G.A. *1. Department of math and computer science Orenburg State*

University

Currently, dynamic processes occurring in the ecosystem of mass open online courses require constant objective assessment, timely adjustment and management [1]. Due to the complexity and independence of learning in an online educational platform, making a forecast of the development of certain events, better known as the regression problem, and evaluating the results achieved, is an important task for practical solutions. Neural networks and gradient boosting are tools for solving this problem. Using the above models, we conducted a series of experiments to predict the performance and delivery date of the planned assessment funds. The results obtained showed (tables 1 and 2) that the proposed algorithms were highly efficient according to the average absolute error criterion. This circumstance was important in the further design of an intelligent online learning platform system that can form individual educational trajectories to improve student performance and ensure optimal time planning of the learning process. Experimental results obtained on different 7 real data sets showed high efficiency of the proposed models.

Table 1-Average absolute error (points) of estimation prediction on 7 data sets

	1	2	3	4	5	6	7
One neural network	9,579	8,870	7,808	7,151	7,717	5,255	7,190
Two neural networks	9,600	8,827	7,760	7,098	7,619	5,325	7,165
Catboost	9,516	8,636	7,685	6,783	7,556	4,957	6,993

Table 2-Average absolute error (days) in predicting the task completion date on 7 data sets

	1	2	3	4	5	6	7
One neural network	1,981	3,552	2,520	2,647	4,141	2,507	2,322
Two neural networks	1,904	3,310	2,497	2,874	4,783	2,592	2,510
Catboost	1,430	2,698	2,134	2,298	5,404	2,075	2,073

As the results showed, the considered models adequately solved the task of predicting learning performance. Based on 7 experiments, the following results of the average absolute error were obtained (table 3).

Table 3-Average absolute errors based on the results of 11 experiments

	Error of 'score' (points)	Error of 'time' (days)
One neural network	7,288	2,412
Two neural networks	7,287	2,459
Catboost	7,122	2,350

Analyzing the data obtained, we can judge that all the models considered solved the problem correctly. It is concluded that the proposed methods can be recommended for use in the design of individual educational trajectories in order to improve student performance and ensure optimal time planning of the learning process.

References

[1] Zaporozhko, V. V. Development of a structural model of mass open online courses based on modern cloud educational platforms / V. V. Zaporozhko, D. I. Parfenov / / Modern science-intensive technologies, 2017. - No. 3. - Pp. 12-17.

[2] Qiu, J., Tang, J., Liu, T., Gong, J. Modeling and predicting learning behavior in MOOCs. In ACM International Conference on Web Search and Data Mining. 2016. pp. 93-102.

[3] Ren, Z. Rangwala, H. Johri, A. Predicting performance on MOOC assessments using multi-regression. In Proceedings of the International Conference Educational Data Mining. 2016. pp. 484-489.

Some typical singularities of the solutions of gas dynamics equations

Shavlukov A. M.

Faculty of Mathematics and Information Technologies
Bashkir State University

In the following paper we have discussed typical umbilical singularity of the formal asymptotic solution of the system of nonlinear equations of a one-dimensional gas flow

$$\rho_T + (\rho v)_X = 0, v_T + v v_X + \alpha(\rho)\rho_X = 0$$

for any smooth pressure function ($\alpha(\rho) > 0$ and doesn't have any singularities at $\rho = 0$) and particular cases for shallow water equations ($\alpha(\rho) = 4$) and cubic gas polythrope (corresponding for $p = A(\rho^3 - \rho_0^3)$, where A, ρ_0 – real constants).

We have put forward a hypothesis that known classification of Riemannian invariants [3] isn't fully correct.

The result was obtained in work with B.I. Suleimanov.

References

[1] Gilmore, Robert. Catastrophe Theory for Scientists and Engineers. New York: Dover, 1993.

[2] Kudashev, V. R. , Suleimanov, B. I. Characteristic features of some typical spontaneous intensity collapse processes in unstable media. Jetp Lett. 62, 382 (1995)

[3] Rakhimov, A.K. Singularities of Riemannian invariants. Funct Anal Its Appl 27, 39 (1993).

On the solvability of the Cauchy problem for some linear functional differential equations

Tahir K.M.T.[1]

Department of functional analysis and its applications
Vladimir State University
the name Alexander Grigorievich and Nikolay Grigorievich Stoletovs

We consider the solvability of the Cauchy problem for some specific linear functional differential equations with delay and neutral type. Formulas are obtained for finding the general solution and the Cauchy function.

Consider a differential equation with constant delay

$$\dot{x}(t) - px(t-1) = f(t), \quad t \geq 0, \quad x(\xi) = 0 \text{ если } \xi < 0, p \in R \tag{1}$$

at primary condition $x(0) = \alpha$.

If $t \in [0,1)$, то $t - 1 \in [-1,0)$, $x(t-1) = 0$, Consequently, we get $\dot{x}(t) - 0 = f(t)$. The solution to this equation will be $x(t) = \alpha + \int_0^t f(s)ds$. The method of mathematical induction can be proved, that for any natural n at $t \in [n, n+1)$ solving the equation (1) is an

$$x(t) = \alpha + \alpha p(t-1) + \frac{\alpha p^2(t-2)^2}{2!} + \ldots + \frac{\alpha p^n(t-n)^n}{n!} + \int_0^t f(s)ds +$$

$$+ p\int_0^{t-1}(t-s-1)f(s)ds + \ldots + p^n\int_0^{t-n}\frac{(t-s-n)^n}{n!}f(s)ds.$$

Thus,

$$x(t) = \sum_{n=0}^{\infty} \chi_{[n,\infty)}(t)\left(\frac{\alpha p^n(t-n)^n}{n!} + \int_0^{t-n}\frac{p^n(t-s-n)^n}{n!}f(s)\,ds\right), \quad t \geq 0. \tag{2}$$

Из (2) we get a representation for the Cauchy function

$$C(t,s) = \sum_{n=0}^{\infty}\frac{p^n(t-s-n)^n\,\chi_{[n,\infty)}(t)\,\chi_{[0,t-n]}(s)}{n!}$$

and fundamental solutions of the corresponding homogeneous equation

$$X(t) = \sum_{n=0}^{\infty}\frac{p^n(t-n)^n\,\chi_{[n,\infty)}(t)}{n!}.$$

Consider a differential equation with delay

$$\dot{x}(t) - px(t/2) = f(t), \quad t \geq 0 \tag{3}$$

under the initial condition $x(0) = \alpha$. Solution of the equation (3) is an

$$x(t) = \alpha\sum_{n=0}^{\infty}\frac{p^n t^n}{n!2^{n(n-1)/2}} + \int_0^t\sum_{n=0}^{\infty}p^n\chi_{[0,t/2^n]}(s)\frac{2^{n(n-1)/2}}{n!}\left(\frac{t}{2^{n-1}} - 2s\right)^n f(s)\,ds. \tag{4}$$

Of (4)we get the representation of the Cauchy function of the equation(3)

$$C(t,s) = \sum_{n=0}^{\infty} p^n \chi_{[0,t/2^n]}(s) \frac{2^{n(n-1)/2}}{n!} \left(\frac{t}{2^{n-1}} - 2s \right)^n$$

and fundamental solution of the corresponding homogeneous equation

$$X(t) = \sum_{n=0}^{\infty} \frac{p^n t^n}{n! 2^{n(n-1)/2}}.$$

The proof of the statements is given in [1], [2].

The work was carried out under the guidance of a professorfunctional analysis and his applicationsVladimir State Universityим. А.Г. и Н.Г.Centennial Родиной Людмилы Ивановны.

References

[1] Жуковский Е.С., Тахир Х.М.Т. Об условиях положительности функции Коши функционально-дифференциальных уравнений // Известия вузов. Математика. 2018. № 11. С. 82–85.

[2] Тахир Х.М.Т. О решении линейных функционально-дифференциальных уравнений // Вестник Тамбовского университета. Серия: Естественные и технические науки. 2016. Т. 21. № 2. С. 417–431.

Scaling entropy sequence for unstable systems

G. Veprev

Department of Mathematics and Computer Science
St. Petersburg State University

The classical notion of KolmogorovSinai entropy is based on dynamics of

measurable partitions of a measure space. For the case of zero entropy sys- tems A.M.Vershik proposed a new approach based on dynamics of functions

of several variables (see [1], [2]). In particular, a measurable metric on the space has some quantitative characteristics wich dynamics yields the new invariant. The works ([3], [4], [5], [6]) of A.M. Vershik, F.V.Petrov and P.B. Zatitskii provide detailed research of this invariant in the case called stable. In this case, the invariant is a class of asymptotically equivalent sequences called the scaling entropy sequence. Until now it was the open question about the existence of unstable systems. The new examples show that there are such spaces. In this case, however, the invariant could be generalized.

References

[1] A. Vershik. Dynamics of metrics in measure spaces and their asymptotic invariants. Markov Processes and Related Fields, 16:1 (2010), 169185.

[2] A. M. Vershik, Scailing entropy and automorphisms with pure pointspec- trum, St. Petersburg Math. J., 23:1 (2012), 75-91

[3] A. Vershik, P. Zatitskiy, F. Petrov. Geometry and dynamics of admis- sible metrics in measure spaces. Open Mathematics, 11 (3), 379400,
2013.

[4] P. B. Zatitskiy, Scaling entropy sequence: invariance and examples, J. Math. Sci. (N. Y.), 209:6 (2015), 890-909

[5] P. B. Zatitskiy, F. V. Petrov, On the subadditivity of a scaling entropy sequence, J. Math. Sci. (N. Y.), 215:6 (2016), 734-737

[6] P. B. Zatitskiy, On the possible growth rate of a scaling entropy se- quence, J. Math. Sci. (N. Y.), 215:6 (2016), 715-733

Dynamics of of flows and homeomorphisms with a finite hyperbolic chain-recurrent set

Zinina S. Kh.

1. Faculty of Mathematics and Information Technology

National Research Mordovia State University

2. Laboratory of Dynamical Systems and Applications

National Research University Higher School of Economics

We investigate general dynamical properties of homeomorphisms and topological flows with a finite hyperbolic chain recurrent set.

Let M^n be a closed n-dimensional manifold with metric d. *A topological flow* on M^n is a family of homeomorphisms $f^t : M^n \to M^n$ that continuously depends on $t \in \mathbf{R}$ and satisfies the following conditions: 1. $f^0(x) = x$ for any point $x \in M^n$; 2. $f^t(f^s(x)) = f^{t+s}(x)$ for any $s, t \in \mathbf{R}$, $x \in M^n$.

The trajectory or the orbit of a point $x \in M^n$ is the set $\mathcal{O}_x = \{f^t(x), t \in \mathbf{R}(\mathbf{Z})\}$.

It is believed that the trajectories of the flow (homeomorphism) are oriented in accordance with an increase in the parameter t. Any two trajectories of a dynamical system either coincide or do not intersect, therefore, the phase space is represented as a union of pairwise disjoint trajectories. There are three types of trajectories:

1) *fixed point* $\mathcal{O}_x = \{x\}$;

2) *periodic trajectory (orbit)* $\mathcal{O}_x$ for which there exists a number $per(x) > 0$ ($per(x) \in N$) such that $f^{per(x)}(x) = x$, but $f^t(x) \neq x$ for all real (natural) numbers $0 < t < per(x)$. The number $per(x)$ is called *period of a periodic orbit* and does not depend on the choice of a point in orbit;

3) *regular trajectory* $\mathcal{O}_x$ — a trajectory that is not a fixed point or a periodic orbit.

To characterize the wandering of the trajectories of a dynamical system, the concept of chain recurrence is traditionally used.

A point $x \in M^n$ is said to be *chain recurrent* for the flow f^t (cascade f), if for any $\varepsilon > 0$ there is $T(n)$, which depends on $\varepsilon > 0$, and there is a ε-chain of the length $T(n)$ from the point x to itself. The set of chain recurrent points of f^t (f) is called the *chain recurrent set* of f^t (f) denoted by $\mathcal{R}_{f^t}$ ($\mathcal{R}_f$).

As a model behavior of flow (homeomorphism) in a neighborhood of a fixed point, we consider a linear flow (homeomorphism) $a_\lambda^t : \mathbf{R}^n \to \mathbf{R}^n$ ($a_\lambda : \mathbf{R}^n \to \mathbf{R}^n$), $\lambda \in \{0, 1, ..., n\}$ of the following form:

$$a_\lambda^t(x_1, ..., x_\lambda, x_{\lambda+1}, ..., x_n) = (2^t x_1, ..., 2^t x_\lambda, 2^{-t} x_{\lambda+1}, ..., 2^{-t} x_n)$$

$$(a_\lambda(x_1, ..., x_\lambda, x_{\lambda+1}, ..., x_n) = (2x_1, ..., 2x_\lambda, 2^{-1} x_{\lambda+1}, ..., 2^{-1} x_n)).$$

A fixed point p of a flow (homeomorphism) f^t (f) is called *is topologically hyperbolic* if there exists a neighborhood $U_p \subset M^n$, the number $\lambda \in \{0, 1, ..., n\}$ and the homeomorphism $h_p : U_p \to \mathbf{R}^n$ such that $h_p f^t|_{U_p} = a_{\lambda_p}^t h_p|_{U_p}$ ($h_p f|_{U_p} = a_{\lambda_p} h_p|_{U_p}$) whenever the left and right sides are defined. Put

$$E_\lambda^s = \{(x_1, ..., x_n) \in \mathbf{R}^n : x_1 = \cdots = x_\lambda = 0\},$$

$$E_\lambda^u = \{(x_1, ..., x_n) \in \mathbf{R}^n : x_{\lambda+1} = \cdots = x_n = 0\}.$$

For a topologically hyperbolic fixed point p of the flow (homeomorphism) f^t (f) of the set $h_p^{-1}(E_{\lambda_p}^s), h_p^{-1}(E_{\lambda_p}^u)$ we will call it *local invariant manifolds*.

The sets

$$W_p^s = \bigcup_{t \in \mathbf{R}} f^t(h_p^{-1}(E_{\lambda_p}^s)), \, W_p^u = \bigcup_{t \in \mathbf{R}} f^t(h_p^{-1}(E_{\lambda_p}^u))$$

will be called *stable and unstable invariant manifolds of the point p*. If the point p is a periodic period of k, then its invariant manifolds are defined as invariant manifolds of the fixed point $f^k(p)$ with respect to the degree of k homeomorphism f.

The number λ_p will be called the *index* of the fixed hyperbolic point p. The index points n and 0 will be called *source* and *sink*, respectively; any point p such that $\lambda_p \in \{1, \cdots, n-1\}$ will be called *saddle*. The number $\lambda_{\mathcal{O}_i}$ will be called the orbit index $\mathcal{O}_i$.

We denote by $\mathcal{F}$ a dynamical system (flow or homeomorphism) with a finite hyperbolic chain-reccurent set $\mathcal{R}_\mathcal{F}$ defined on M^n. For an orbit flow, the sets $\mathcal{R}_\mathcal{F}$ coincide with fixed points. We introduce a class of dynamical systems G consisting of systems of type $\mathcal{F}$.

The dynamics of systems of this class are close in their properties to gradient-like systems. Namely, similar to S. Smale's order, we introduce a partial order relation on the set of chain-recurrent orbits of the dynamical system $\mathcal{F}$ by the condition: $\mathcal{O}_i \prec \mathcal{O}_j \iff W_{\mathcal{O}_i}^s \cap W_{\mathcal{O}_j}^u \neq \emptyset$, where $\mathcal{O}_i, \mathcal{O}_j$ – orbits from the set $\mathcal{R}_\mathcal{F}$, $W_{\mathcal{O}_i}^s = \bigcup_{p \in \mathcal{O}_i} W_p^s$, $W_{\mathcal{O}_i}^u = \bigcup_{p \in \mathcal{O}_i} W_p^u$.

The dynamical system $\mathcal{F}$ has no cycles; therefore, the introduced relation can be continued (not uniquely) to a relation of full order, i. e. for any $\mathcal{O}_i, \mathcal{O}_j$ either $\mathcal{O}_i \prec \mathcal{O}_j$, or $\mathcal{O}_j \prec \mathcal{O}_i$ on $\mathcal{R}_\mathcal{F}$. In what follows, we will consider the orbits of the system $\mathcal{F}$ numbered in accordance with the introduced order:

$$O_1 \prec \cdots \prec \mathcal{O}_k.$$

In addition, without loss of generality, we assume that any sink orbit is located in this order below any saddle orbit, and any source orbit is higher than any saddle one.

For systems of class G the following result is proved.

Theorem.

Let $\mathcal{F} \in G(M^n)$. Then

1. $M^n = \bigcup_{i=1}^k W_{\mathcal{O}_i}^u = \bigcup_{i=1}^k W_{\mathcal{O}_i}^s$;

2. $W_{\mathcal{O}_i}^u$ ($W_{\mathcal{O}_i}^s$) s a topological submanifold of M^n, homeomorphic to $\mathbf{R}^{\lambda_{\mathcal{O}_i}}$ ($\mathbf{R}^{n-\lambda_{\mathcal{O}_i}}$);

3. $cl(W_{\mathcal{O}_i}^u) \setminus (W_{\mathcal{O}_i}^u \cup \mathcal{O}_i) \subset \bigcup_{j=1}^{i-1} W_{\mathcal{O}_j}^u$ ($cl(W_{\mathcal{O}_i}^s) \setminus (W_{\mathcal{O}_i}^s \cup \mathcal{O}_i) \subset \bigcup_{j=i+1}^k W_{\mathcal{O}_j}^s$).