

**On-line «Master in Computer Vision» educational program.  
Entrance examination test**

**Variant 2.**

1. Find all pairs  $(a, b)$  of positive integer numbers  $a$  and  $b$ , such that, the following system of equations has infinitely many solutions

$$ax_1 + 6x_2 = 2$$

$$2x_1 + bx_2 = 1$$

2. A simple, undirected graph is given by its adjacency matrix.

- find the degrees of the vertices of the graph, and the diameter of the graph
- show that this graph has no Euler cycle
- what is the minimum number of edges (and exactly which edges) to add to the graph for the Euler cycle to appear.

0 1 0 1 1 0

1 0 1 1 0 0

0 1 0 0 1 1

1 1 0 0 0 1

1 0 1 0 0 0

0 0 1 1 0 0

3. Cumulative distribution function for random variable  $X$  has the form

$$F(x) = (3x^2 - x^3)/4 \text{ on } [0; 2] \text{ and } F(x) = 0, x < 0, F(x) = 1, x > 2.$$

- find the probability density function of  $X$ . Draw the graph of it.
- calculate the expected value (mathematical expectation) of  $X$ .
- For the random variable  $Y = 2X - 1$ , find the probability  $P(1 < Y < 5)$ .

In the following problem it is necessary to suggest the most efficient algorithms. Full points are given for the most efficient algorithm having the lowest computational complexity. The lower the efficiency of the suggested solution, the lower are the points. For example, for two problems below 10 points are given for an algorithm of complexity  $O(n)$ , 7 points are given for an algorithm of complexity  $O(n \log n)$ , and 4 points are given for an algorithm of complexity  $O(n^2)$

4. Write a pseudo-code (or code on any programming language) of an algorithm, which find two elements in an array of integer numbers (positive and negative), such that the sum of these elements is maximal. Discuss the computational complexity of your algorithm.