НАЦИОНАЛЬНЫЙ ИССЛЕДОВАТЕЛЬСКИЙ УНИВЕРСИТЕТ

Воронежский государственный университет

К вопросу о корректности численного метода FDTD в применении к электродинамическому моделированию радиоэлементов

А. А. Кононов, Г. К. Усков, С.П. Скулкин

МИНОБРНАУКИ РОССИИ

ФЕДЕРАЛЬНОЕ ГОСУДАРСТВЕННОЕ БЮДЖЕТНОЕ ОБРАЗОВАТЕЛЬНОЕ УЧРЕЖДЕНИЕ ВЫСШЕГО ОБРАЗОВАНИЯ «ВОРОНЕЖСКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ»

Физический факультет

Кафедра электроники

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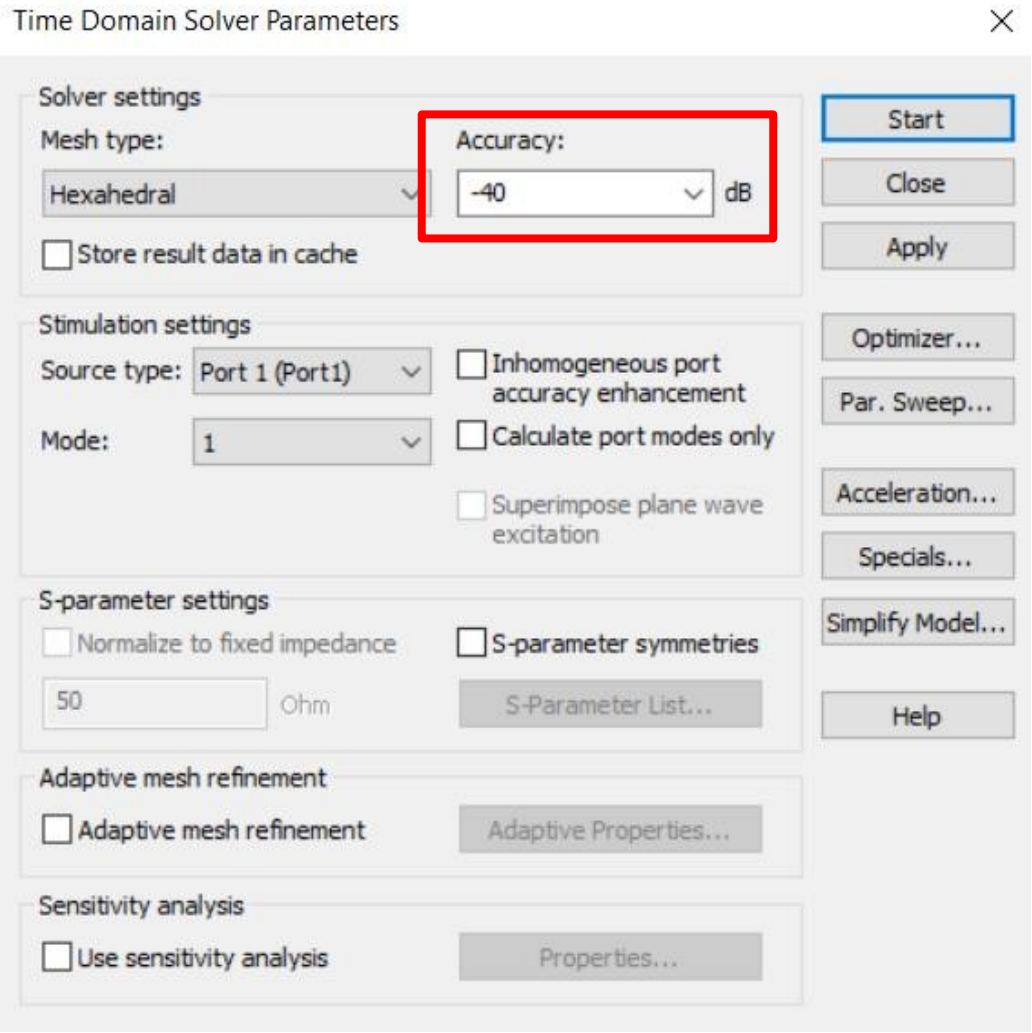
Направление 03.04.03 Радиофизика

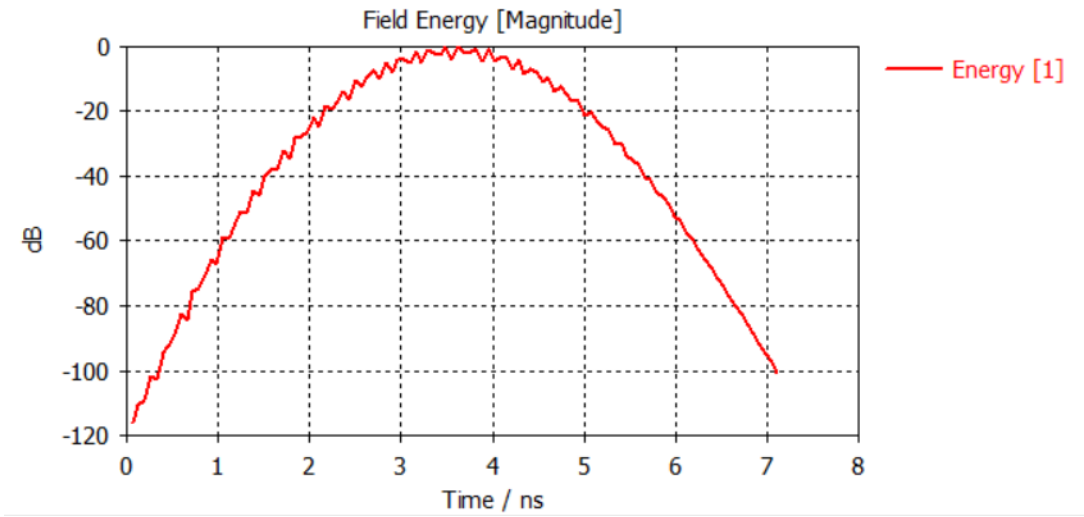
Профиль Системы телекоммуникаций и радиоэлектронной борьбы

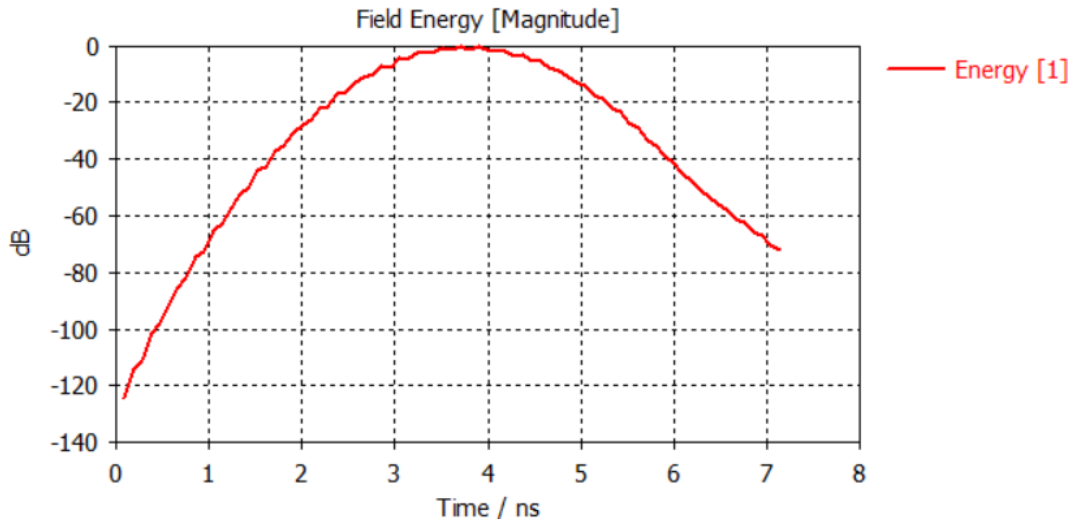
Подготовил: Кононов А. А.

Руководитель: д. ф.-м. н., доцент Усков Г. К.

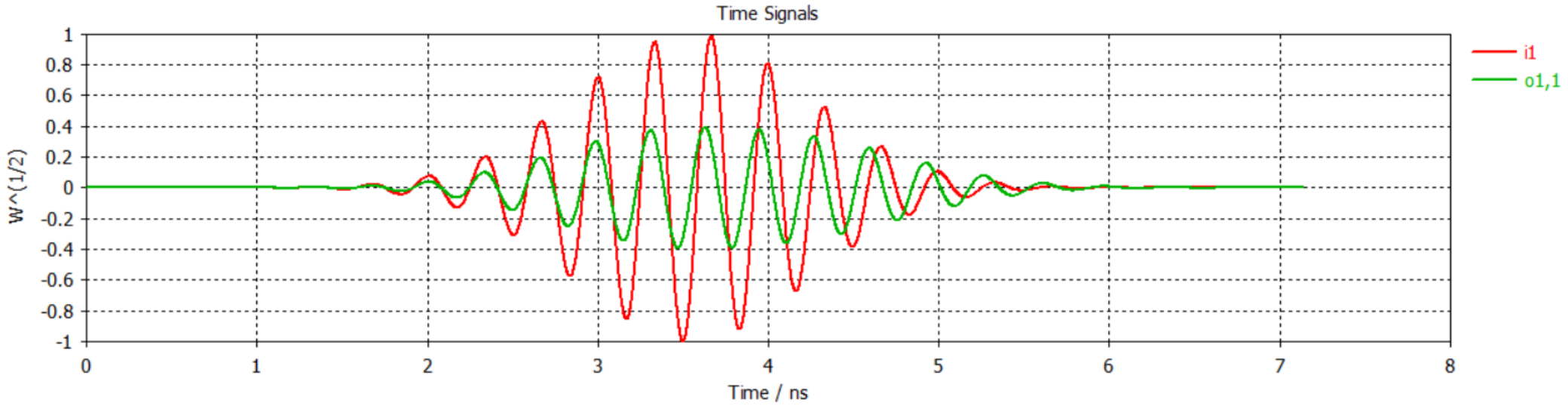
Энергетический критерий остановки

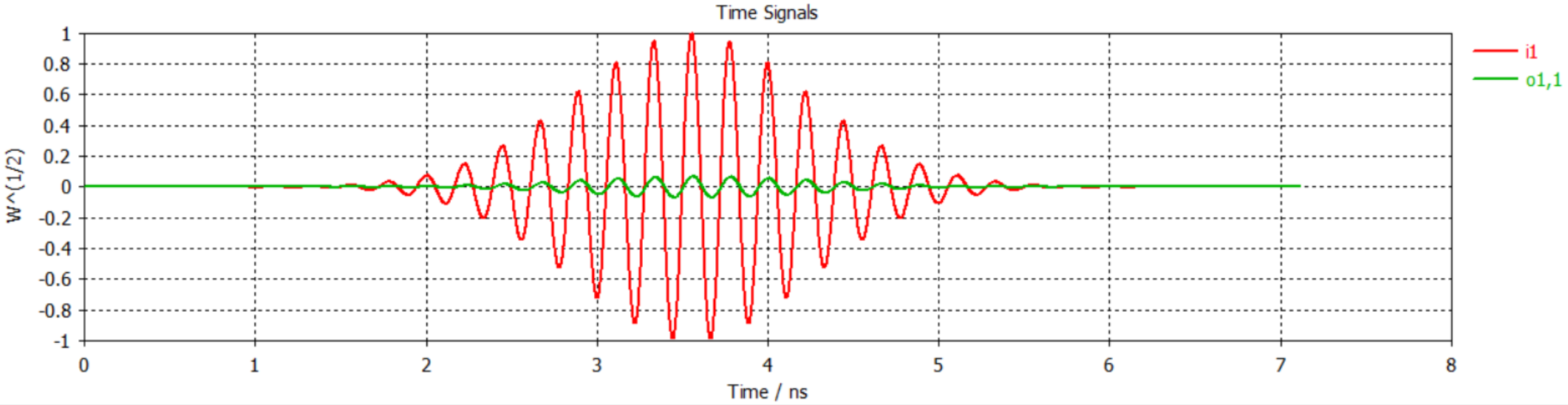




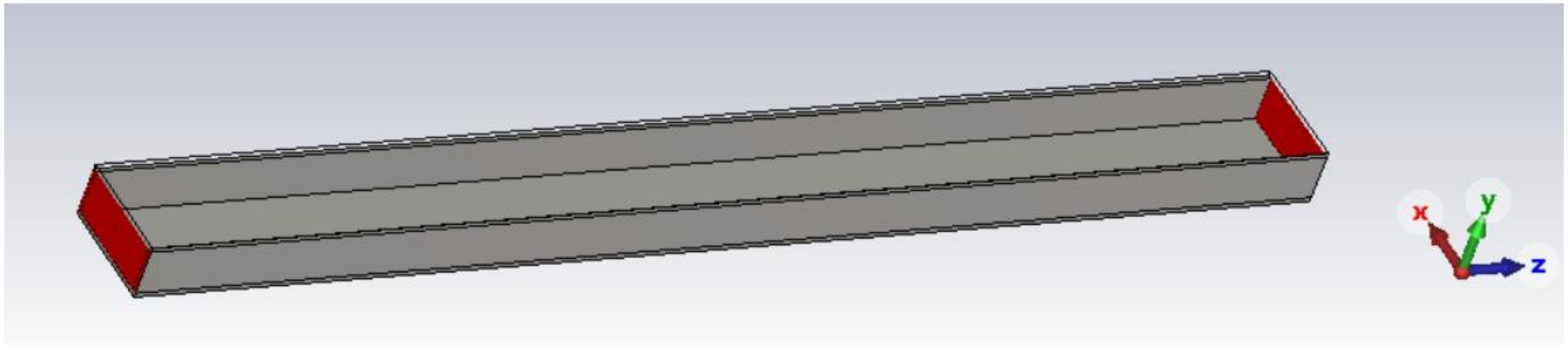


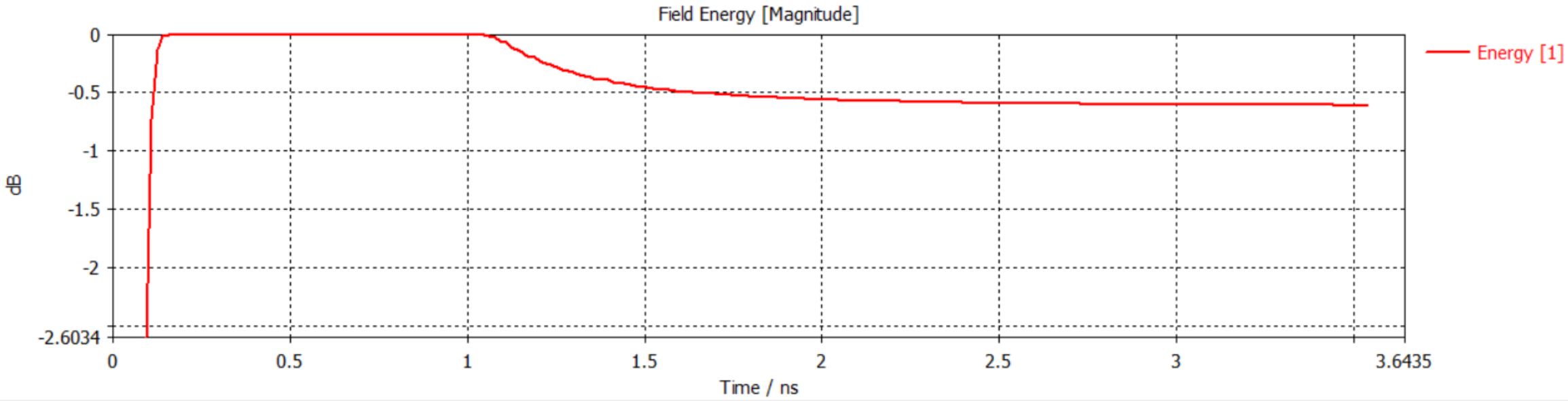
Сигналы во временно́й области



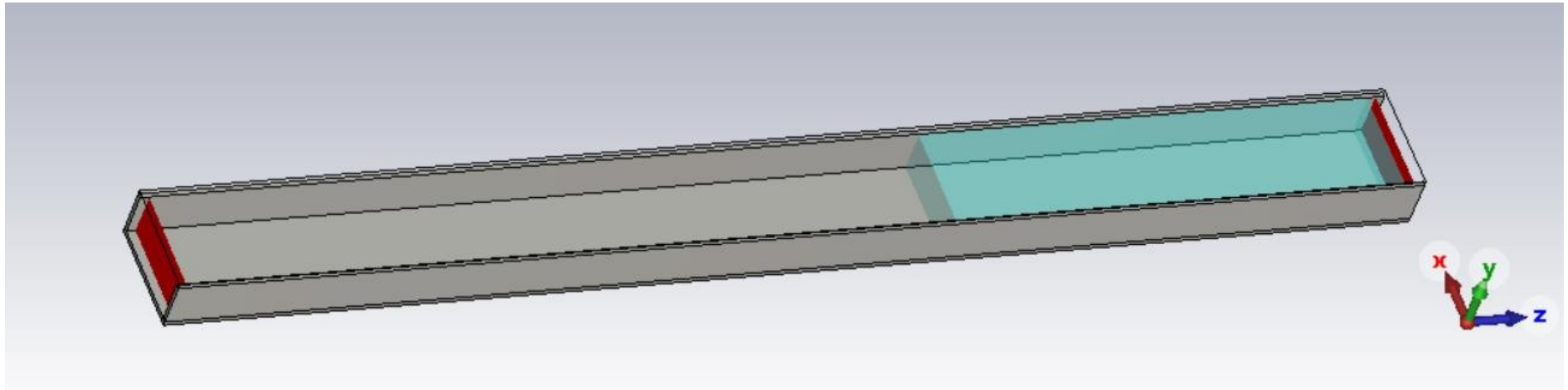


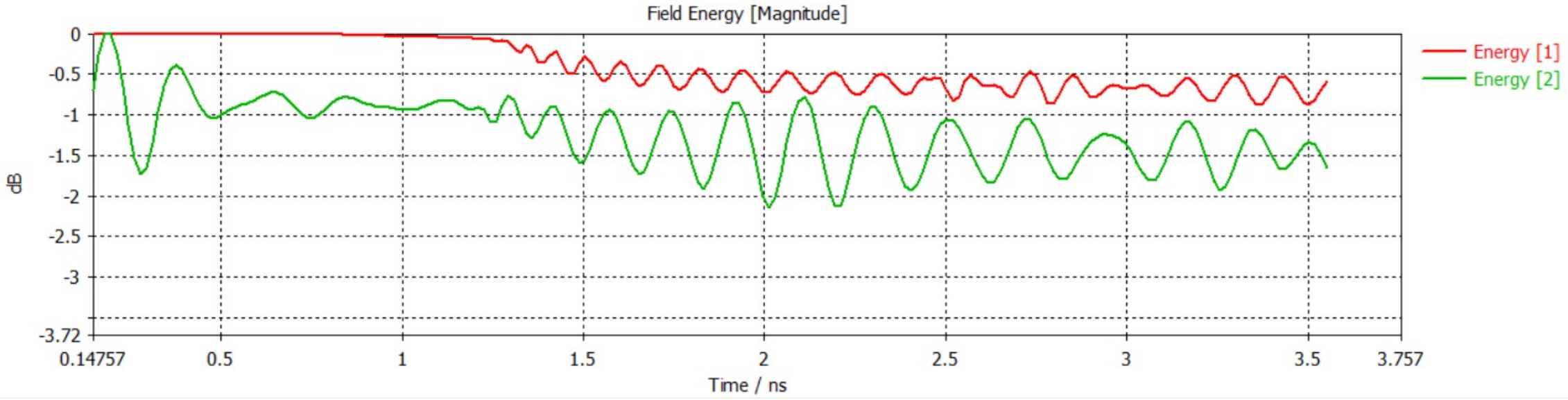
Сбой критерия остановки — 1



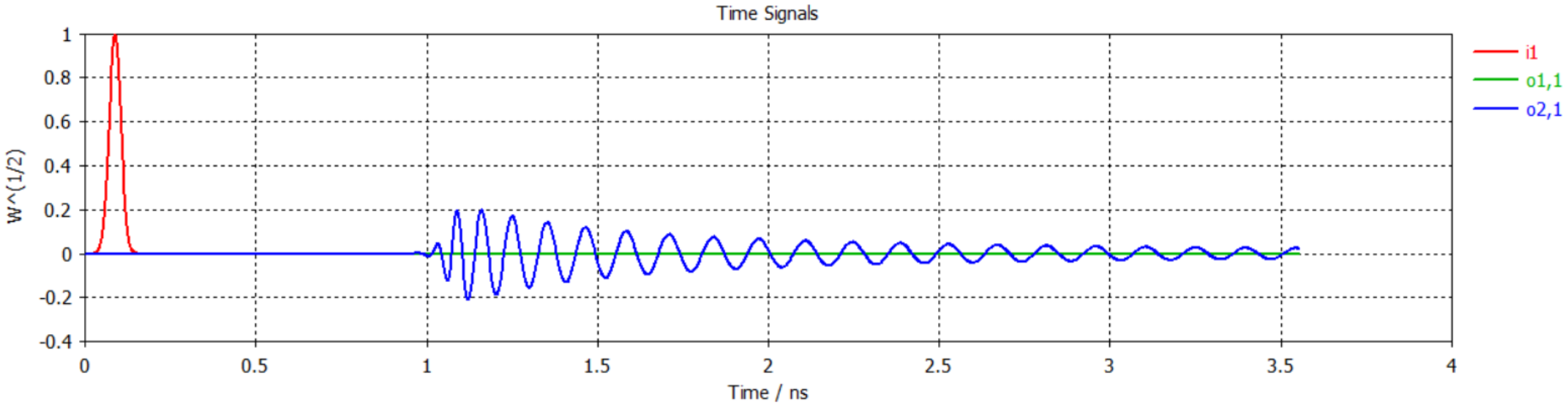


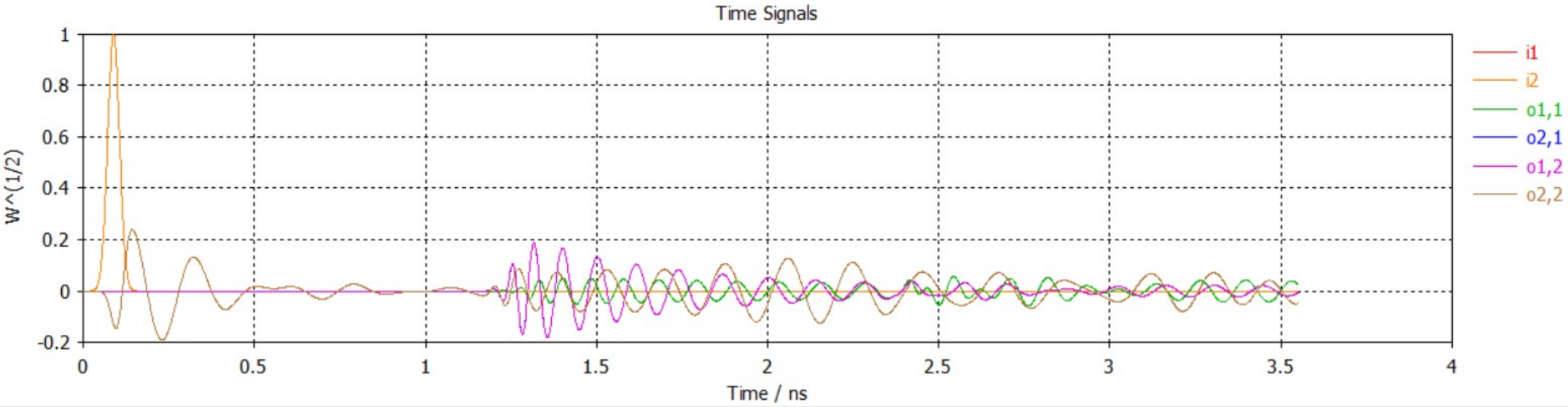
Сбой критерия остановки — 2



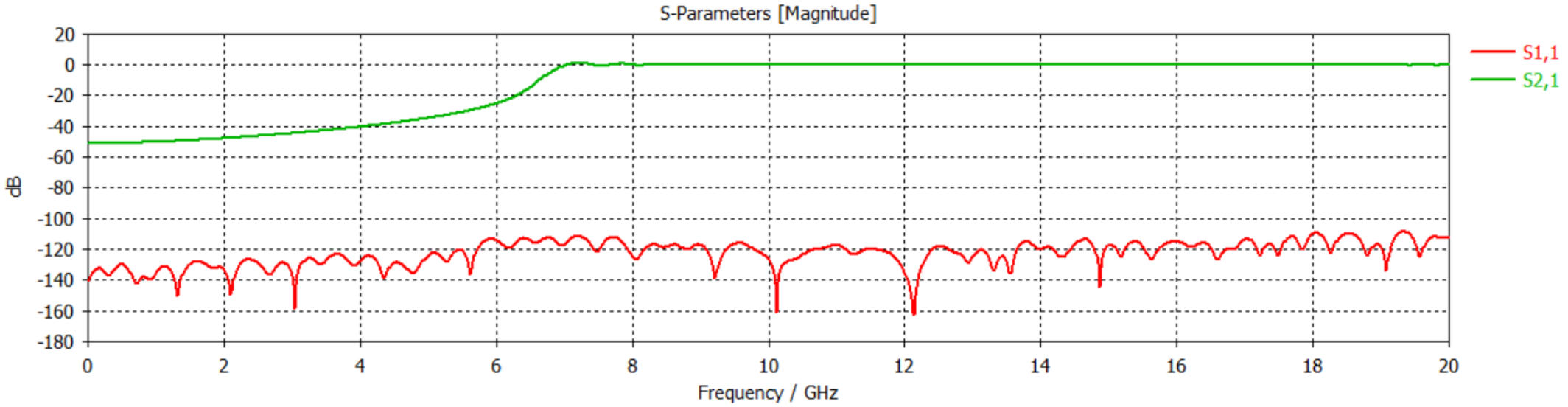


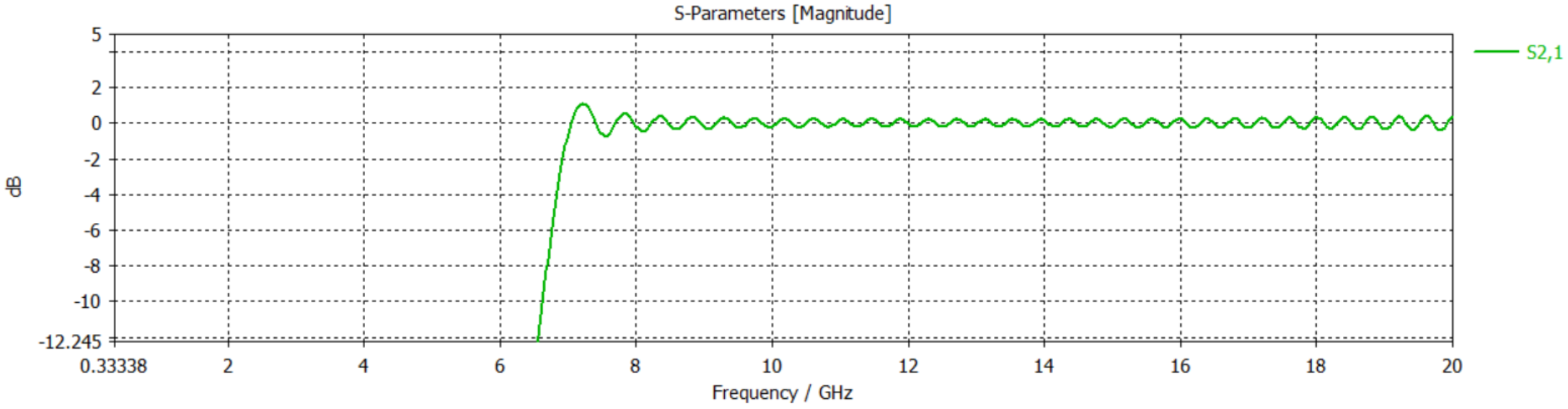
Последствия во временно́й области



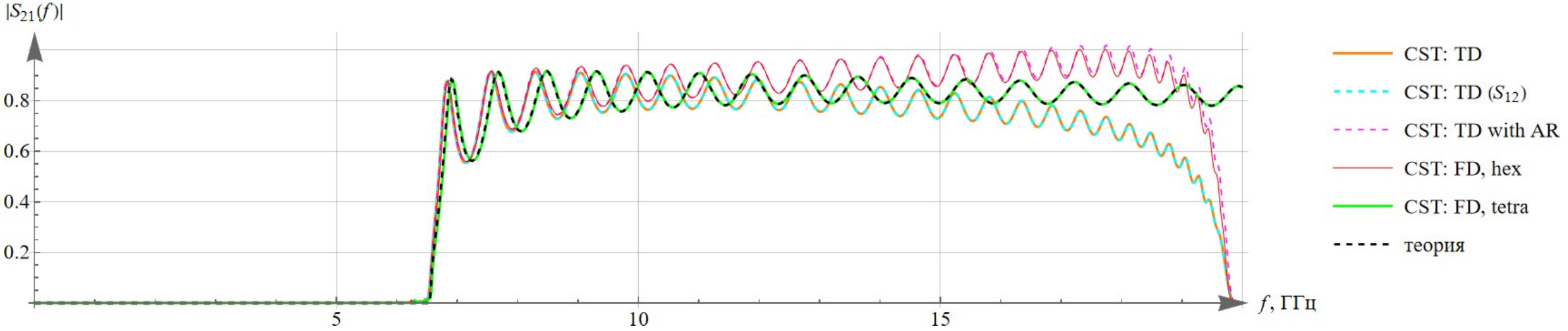


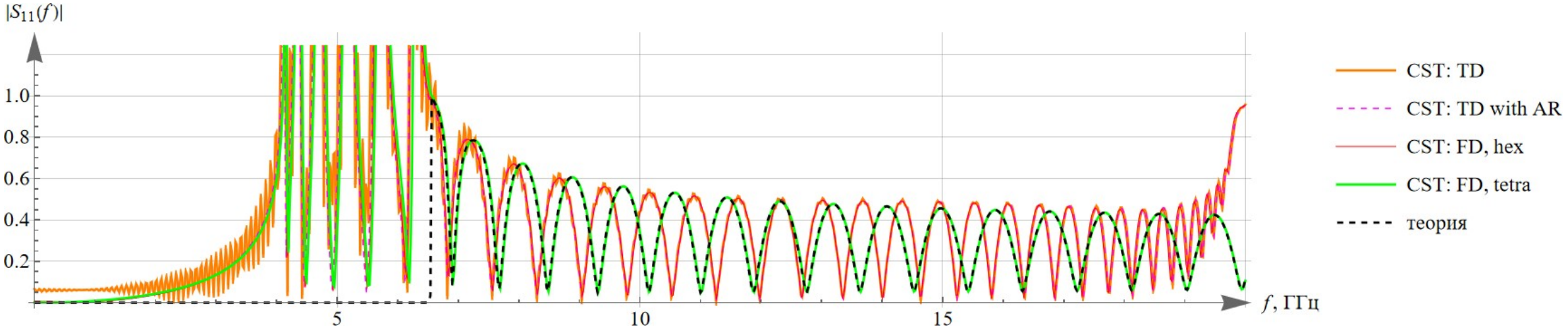
Ошибки в *S*-параметрах пустого волновода



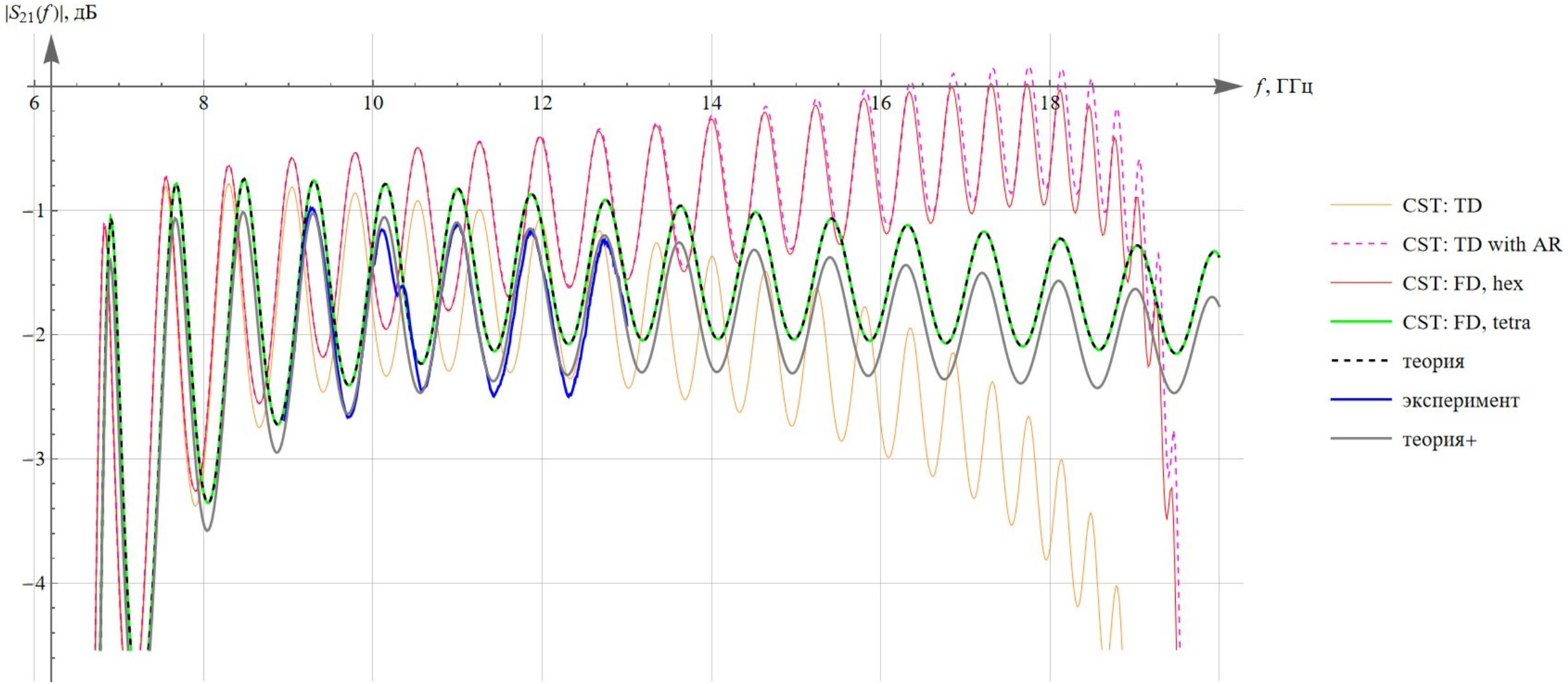


Ошибки для волновода с диэлектриком





Ошибка для $|S_{21}|_{[\text{дБ}]}$ в рабочем диапазоне



Обобщённое волновое уравнение

$$k = \omega\sqrt{\varepsilon\varepsilon_0\mu\mu_0},$$
$$\hat{\varepsilon} = \varepsilon\left(1 - i\,\mathrm{tg}\,\mathrm{P}\right),$$
$$\hat{k} = \omega\sqrt{\hat{\varepsilon}\varepsilon_0\mu\mu_0}$$

$$\mathrm{rot}\,\hat{\vec{E}} = -i\omega\hat{\vec{B}}, \quad \mathrm{rot}\,\hat{\vec{H}} = \hat{\vec{j}} + i\omega\hat{\vec{D}} \qquad \left(\vec{\nabla}\cdot\hat{\vec{D}}\right) = \hat{\rho}, \qquad \left(\vec{\nabla}\cdot\hat{\vec{B}}\right) = 0$$

$$\hat{\vec{D}}(\vec{r}) = \varepsilon_0\varepsilon(\vec{r})\hat{\vec{E}}(\vec{r}), \qquad \hat{\vec{B}}(\vec{r}) = \mu_0\mu(\vec{r})\hat{\vec{H}}(\vec{r}), \qquad \hat{\vec{j}}(\vec{r}) = \sigma(\vec{r})\hat{\vec{E}}(\vec{r}) + \hat{\vec{j}}_e(\vec{r})$$

$$\omega^2\mu_0\mu\varepsilon_0\varepsilon\vec{E} + \Delta\vec{E} + \left(\vec{\nabla}\ln\varepsilon\cdot\vec{\nabla}\right)\hat{\vec{E}} + \left(\hat{\vec{E}}\cdot\vec{\nabla}\ln\varepsilon\right)\vec{\nabla}\ln\varepsilon + \left[\vec{\nabla}\ln\varepsilon\mu\times\left[\vec{\nabla}\times\hat{\vec{E}}\right]\right] = i\omega\mu_0\mu\hat{\vec{j}} + \frac{1}{\varepsilon_0}\vec{\nabla}\frac{\hat{\rho}}{\varepsilon}$$

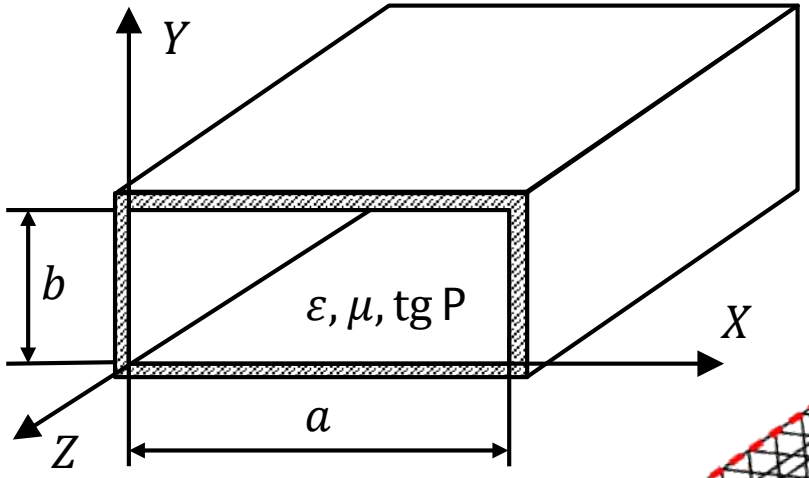
$$j\omega\hat{\rho} + \left(\vec{\nabla}\cdot\hat{\vec{j}}\right) = 0 \Longrightarrow \hat{\rho} = \frac{\sigma\left(\hat{\vec{E}}\cdot\nabla\ln\mathrm{tg}\,\mathrm{P}\right) + \left(\vec{\nabla}\cdot\hat{\vec{j}}_a\right)}{\omega(j + \mathrm{tg}\,\mathrm{P})} \Longrightarrow \frac{1}{\varepsilon_0}\vec{\nabla}\frac{\hat{\rho}}{\varepsilon} = j\vec{\nabla}\left(\frac{\left(\hat{\vec{E}}\cdot\nabla\,\mathrm{tg}\right) + \frac{\mathrm{tg}\,\mathrm{P}}{\sigma}\left(\vec{\nabla}\cdot\hat{\vec{j}}_a\right)}{\varepsilon\,(1 - j\,\mathrm{tg}\,\mathrm{P})}\right)$$

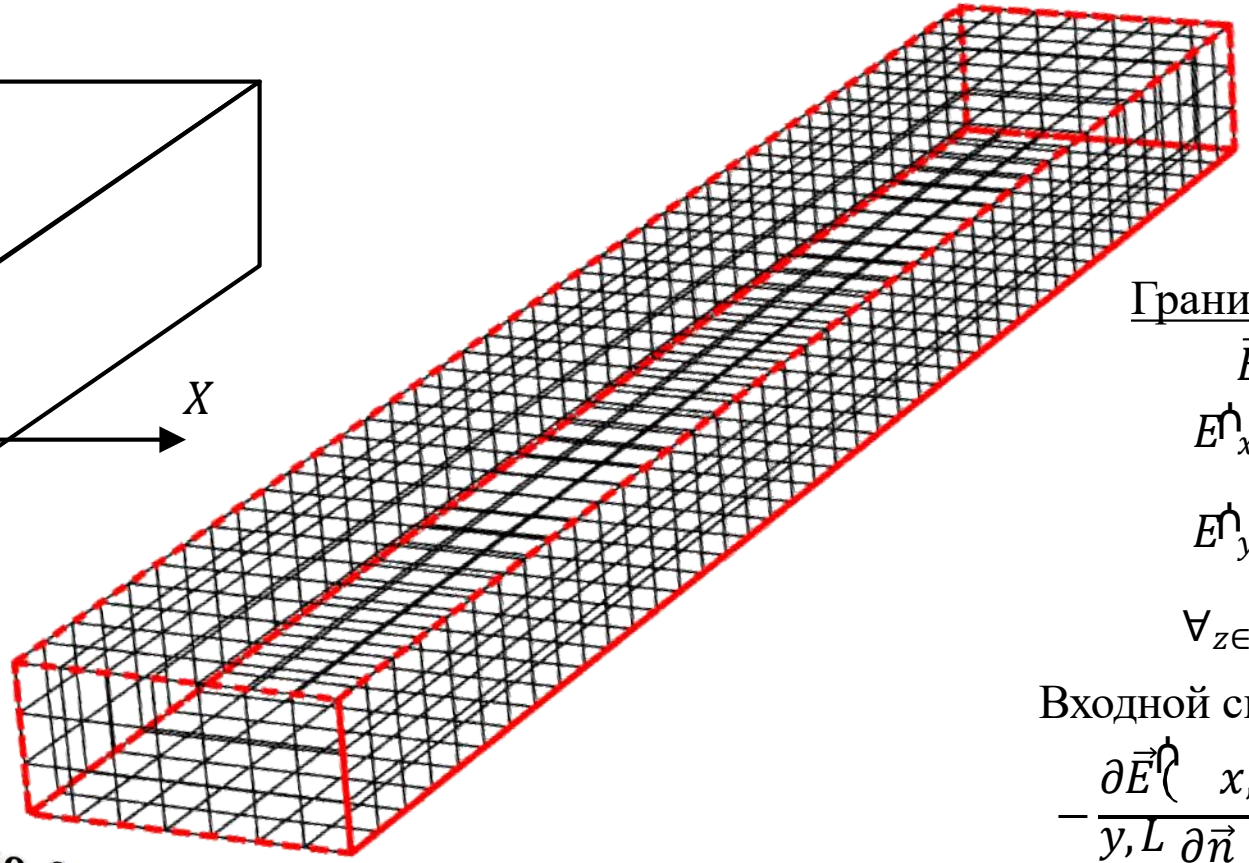
$$\hat{\vec{j}}_a \equiv \vec{0}$$
$$\mu \equiv const$$

$$\hat{k}\hat{\vec{E}} + \Delta\hat{\vec{E}} + \vec{\nabla}\left(\hat{\vec{E}}\cdot\vec{\nabla}\ln\varepsilon\right) - \left[\left[\vec{\nabla}\times\hat{\vec{E}}\right]\times\vec{\nabla}\ln\mu\right] = j\omega\mu_0\mu\hat{\vec{j}}_e + j\vec{\nabla}\left(\frac{\left(\vec{\nabla}\cdot\hat{\vec{j}}_a\right)}{\omega\varepsilon_0\varepsilon}\right)$$

$$\omega^2\varepsilon(\vec{r})\mu\mu_0\hat{\vec{E}}(\vec{r}) + \vec{\nabla}^2\hat{\vec{E}}(\vec{r}) + \vec{\nabla}\left(\hat{\vec{E}}(\vec{r})\cdot\vec{\nabla}\ln\varepsilon(\vec{r})\right) = 0$$

Модель пустого волновода





Граничные условия:

$$\vec{E}_t \equiv \vec{0} \Longrightarrow$$

$$E_x\big|_{y\in\{0;b\}} \equiv 0,$$

$$E_y\big|_{x\in\{0;a\}} \equiv 0,$$

$$\forall_{z\in(0;L)}\ E_z \equiv 0$$

Входной сигнал: $\vec{E}_{in}(x,y) \Longrightarrow$

$$-\frac{\partial \vec{E}(x,y,L)}{\partial \vec{n}} = i\beta \vec{E}(x,y,L),$$

$$-\frac{\partial \vec{E}(x,y,0)}{\partial \vec{n}} = i\beta\left(\vec{E}(x,y,0) - 2\vec{E}_{in}(x,y)\right).$$

```
reg1
= ToElementMesh[Cuboid[{0, 0, 0}, {a, b, L1}], "MaxCellMeasure"
→ 0.00000005, "MeshElementType" → HexahedronElement]"
ElementMesh[{{0., 0.02286}, {0., 0.01016}, {0., 0.15}}, {HexahedronElement[⟨861⟩]}]
```

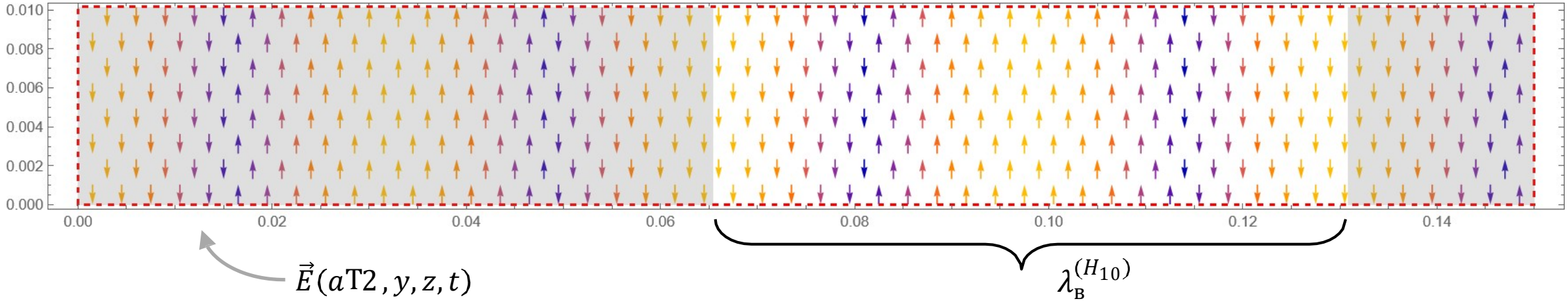
Электрическое поле волны типа H_{10}

$$f = 8.0\ \text{ГГц}, \qquad a = 22.86\ \text{мм}, \qquad b = 10.16\ \text{мм}, \qquad L = 150\ \text{мм}$$

$$\varepsilon_{\text{в}} = 1.00059, \qquad \operatorname{tg} \mathrm{P}_{\text{в}} = 1 \cdot 10^{-15}, \qquad \mu_{\text{в}} = 1$$

$$E(z) \sim e^{-j\beta z}, \qquad \beta \approx \frac{2\pi}{\lambda_{\text{в}}}$$

$$\lambda_{\text{в}}^{(H_{10})} = \frac{\lambda_{\text{c}}}{\sqrt{1 - \left(\lambda_{\text{c}}^{(H_{10})}/\lambda_{\text{кр}}\right)^2}} \approx 65.4\ \text{мм}, \qquad \lambda_{\text{c}} = \frac{v_{\text{c}}}{f} = \frac{1}{f\sqrt{\varepsilon_{\text{в}}\varepsilon_0\mu_{\text{в}}\mu_0}} \approx 37.5\ \text{мм}, \qquad \lambda_{\text{кр}}^{(H_{10})} = 2a \approx 45.7\ \text{мм}$$



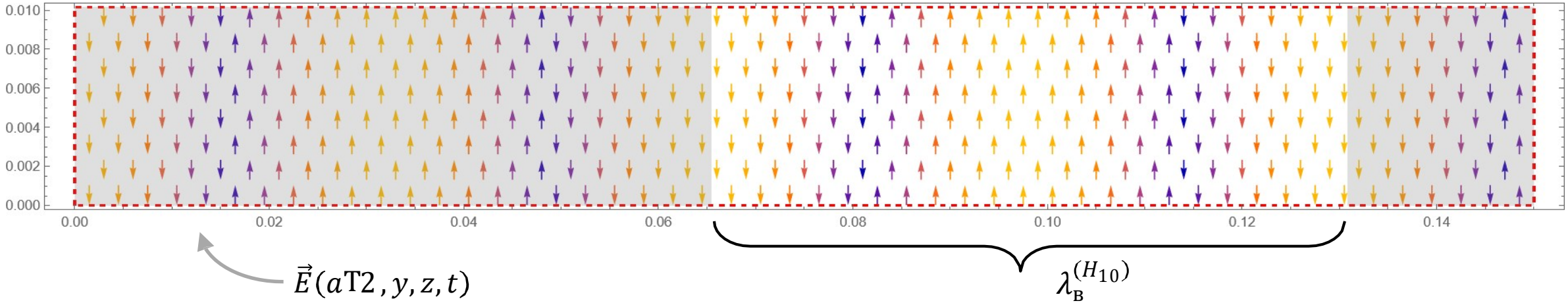
Распространение волны типа H_{10}

$$f = 8.0\ \text{ГГц}, \qquad a = 22.86\ \text{мм}, \qquad b = 10.16\ \text{мм}, \qquad L = 150\ \text{мм}$$

$$\varepsilon_{\text{в}} = 1.00059, \qquad \operatorname{tg} \mathrm{P}_{\text{в}} = 1 \cdot 10^{-15}, \qquad \mu_{\text{в}} = 1$$

$$E(z) \sim e^{-j\beta z}, \qquad \beta \approx \frac{2\pi}{\lambda_{\text{в}}}$$

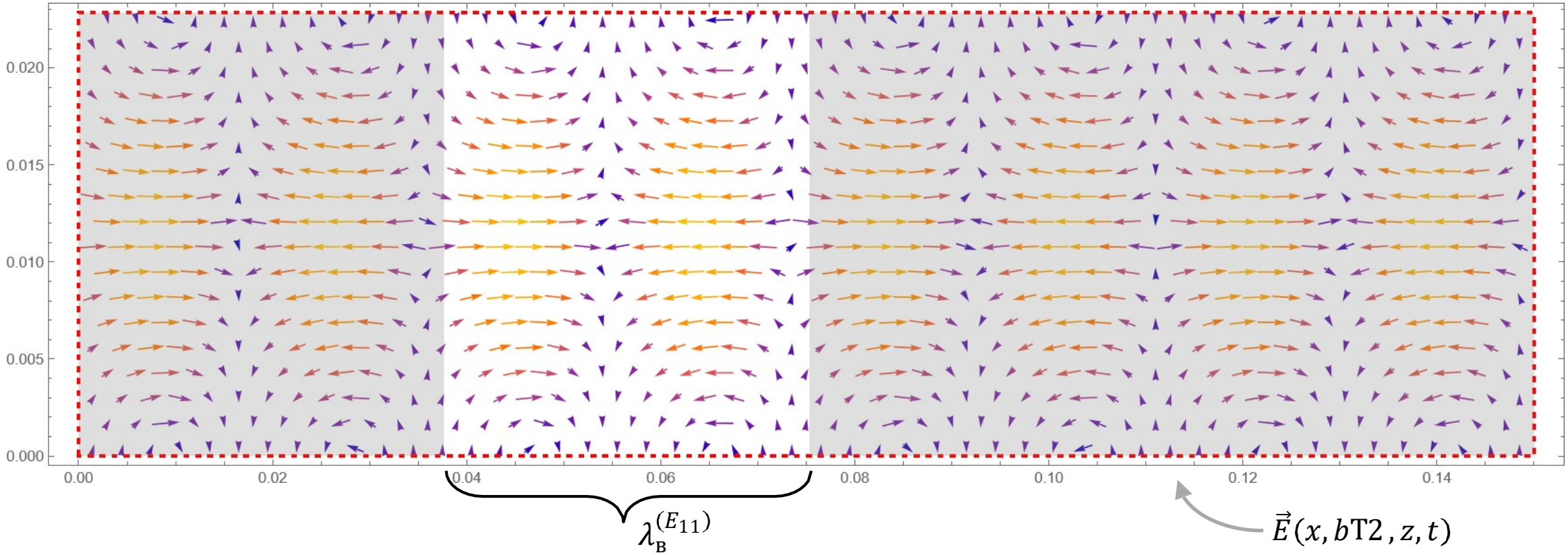
$$\lambda_{\text{в}}^{(H_{10})} = \frac{\lambda_{\text{c}}}{\sqrt{1 - \left(\lambda_{\text{c}}^{(H_{10})}_{\text{кр}}\right)^2}} \approx 65.4\ \text{мм}, \qquad \lambda_{\text{c}} = \frac{v_{\text{c}}}{f} = \frac{1}{f\sqrt{\varepsilon_{\text{в}}\varepsilon_0\mu_{\text{в}}\mu_0}} \approx 37.5\ \text{мм}, \qquad \lambda_{\text{кр}}^{(H_{10})} = 2a \approx 45.7\ \text{мм}$$



Электрическое поле волны типа E_{11}

$f = 18.0$ ГГц, $a = 22.86$ мм, $b = 10.16$ мм, $L = 150$ мм

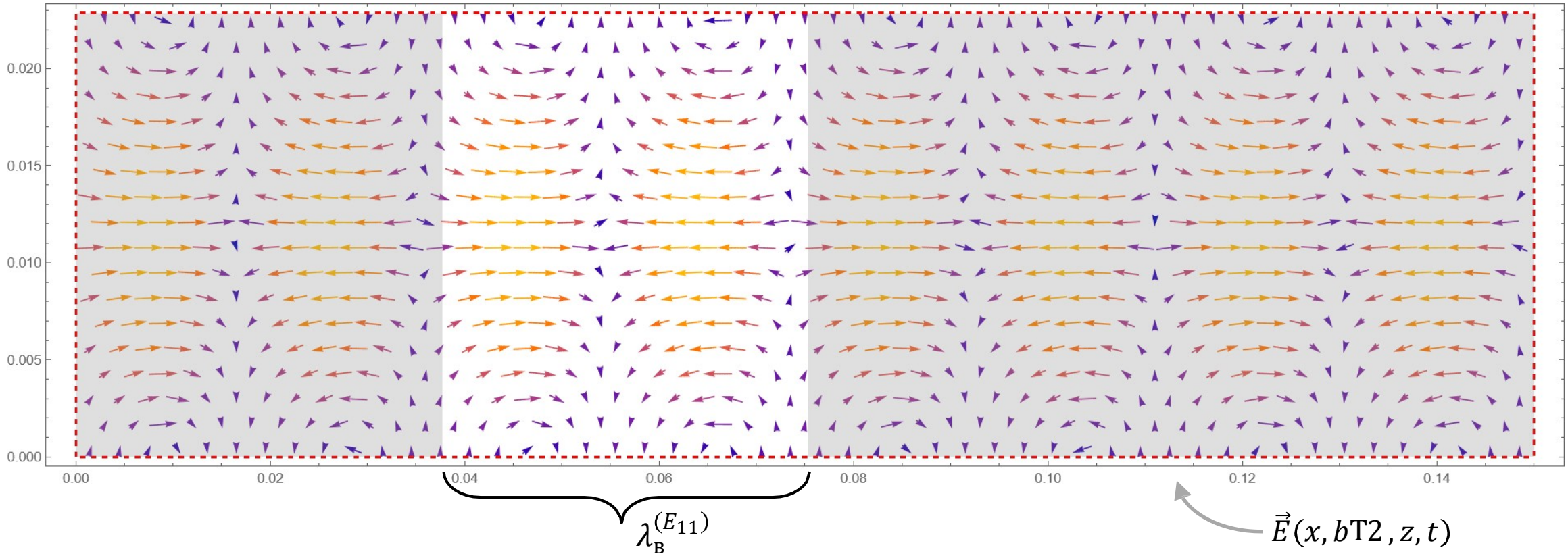
$$\lambda_{\text{в}}^{(E_{11})} = \frac{\lambda_{\text{с}}}{\sqrt{1 - \left(\lambda_{\text{с}}\text{T}\lambda_{\text{кр}}^{(E_{11})}\right)^2}} \approx 37.7 \text{ мм}, \qquad \lambda_{\text{с}} = \frac{v_{\text{с}}}{f} = \frac{1}{f\sqrt{\varepsilon\varepsilon_0\mu\mu_0}} \approx 16.7 \text{ мм}, \qquad \lambda_{\text{кр}}^{(E_{11})} = \frac{2}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}} \approx 18.6 \text{ мм}$$



Распространение волны типа E_{11}

$f = 18.0$ ГГц, $a = 22.86$ мм, $b = 10.16$ мм, $L = 150$ мм

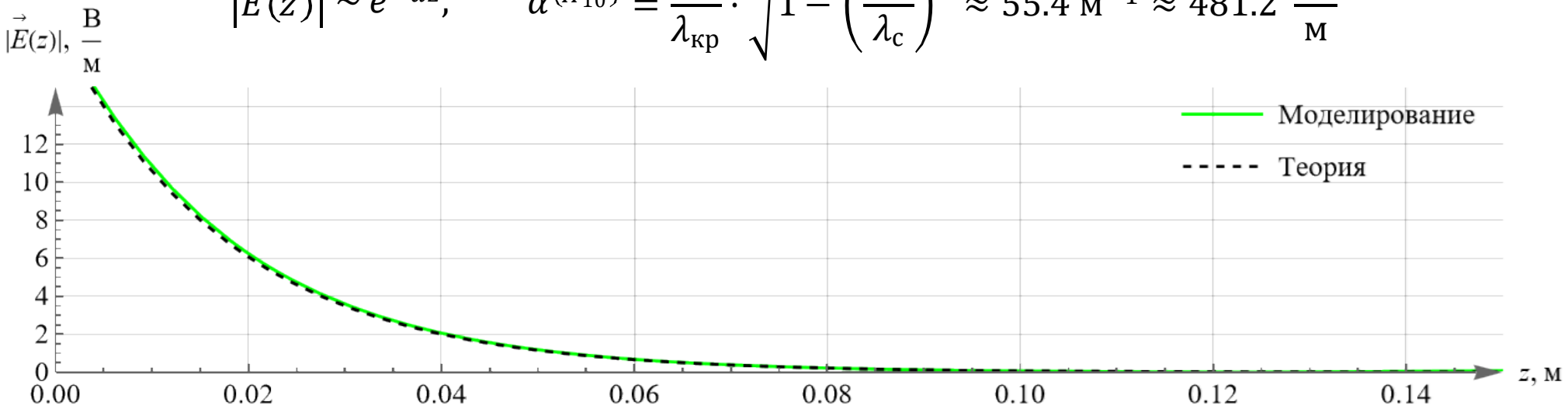
$$\lambda_{\text{в}}^{(E_{11})} = \frac{\lambda_{\text{с}}}{\sqrt{1 - \left(\lambda_{\text{с}}\text{T}\lambda_{\text{кр}}^{(E_{11})}\right)^2}} \approx 37.7 \text{ мм}, \qquad \lambda_{\text{с}} = \frac{v_{\text{с}}}{f} = \frac{1}{f\sqrt{\varepsilon\varepsilon_0\mu\mu_0}} \approx 16.7 \text{ мм}, \qquad \lambda_{\text{кр}}^{(E_{11})} = \frac{2}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}} \approx 18.6 \text{ мм}$$

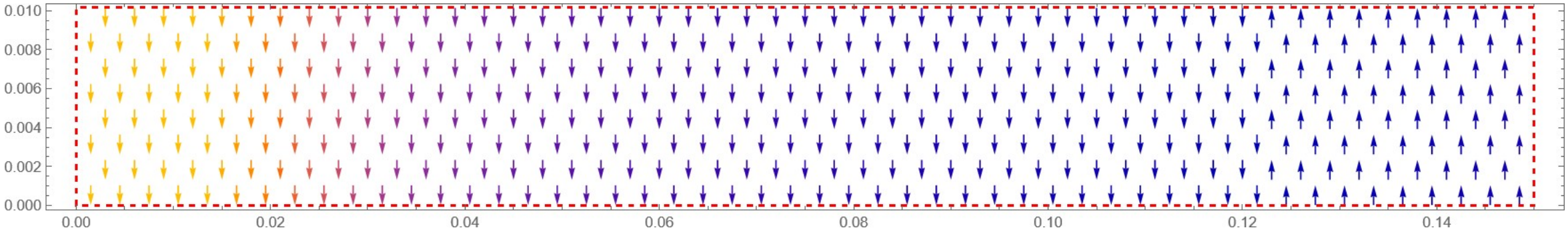


Запредельный волновод (отсечка моды H_{10})

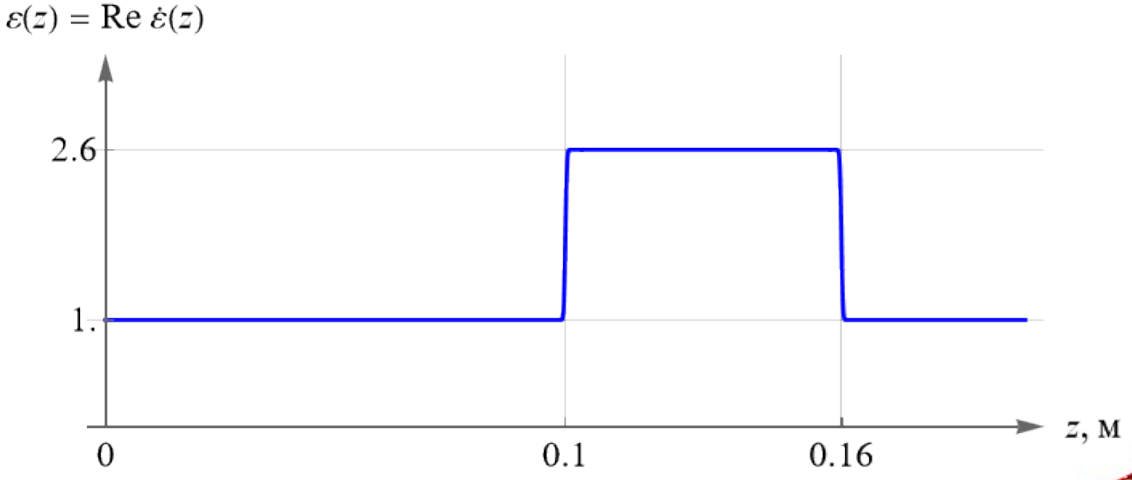
$f = 6.0$ ГГц, $a = 22.86$ мм, $b = 10.16$ мм, $L = 150$ мм

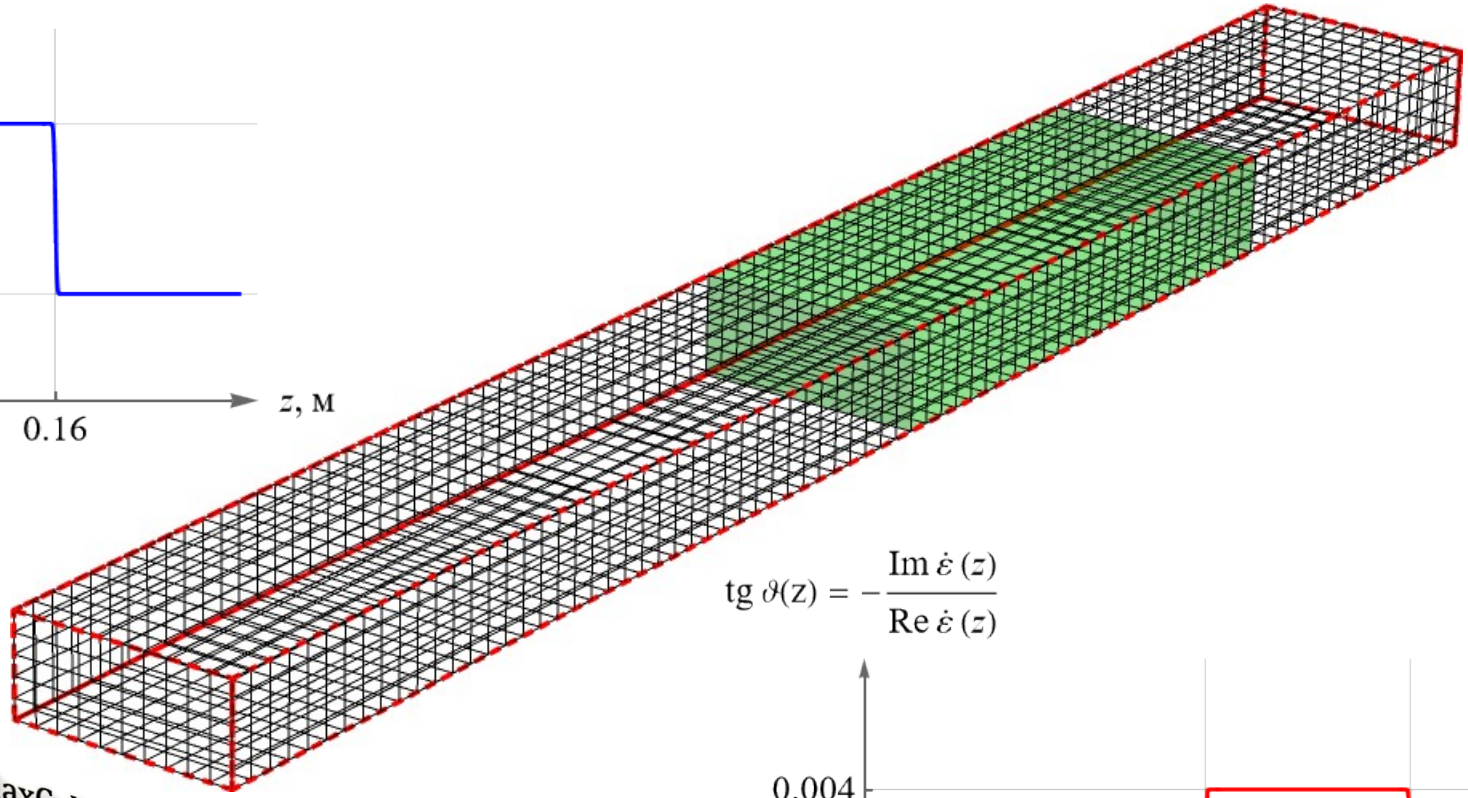
$$\left|\vec{E}(z)\right| \sim e^{-\alpha z}, \qquad \alpha^{(H_{10})} = \frac{2\pi}{\lambda_{\text{кр}}} \cdot \sqrt{1 - \left(\frac{\lambda_{\text{кр}}}{\lambda_{\text{с}}}\right)^2} \approx 55.4\ \text{м}^{-1} \approx 481.2\ \frac{\text{дБ}}{\text{м}}$$



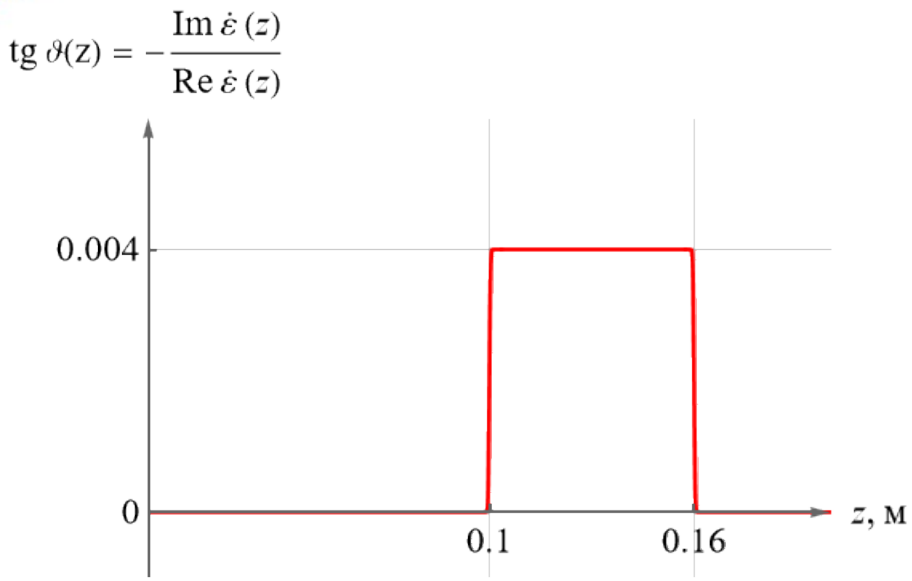


Модель волновода с диэлектриком



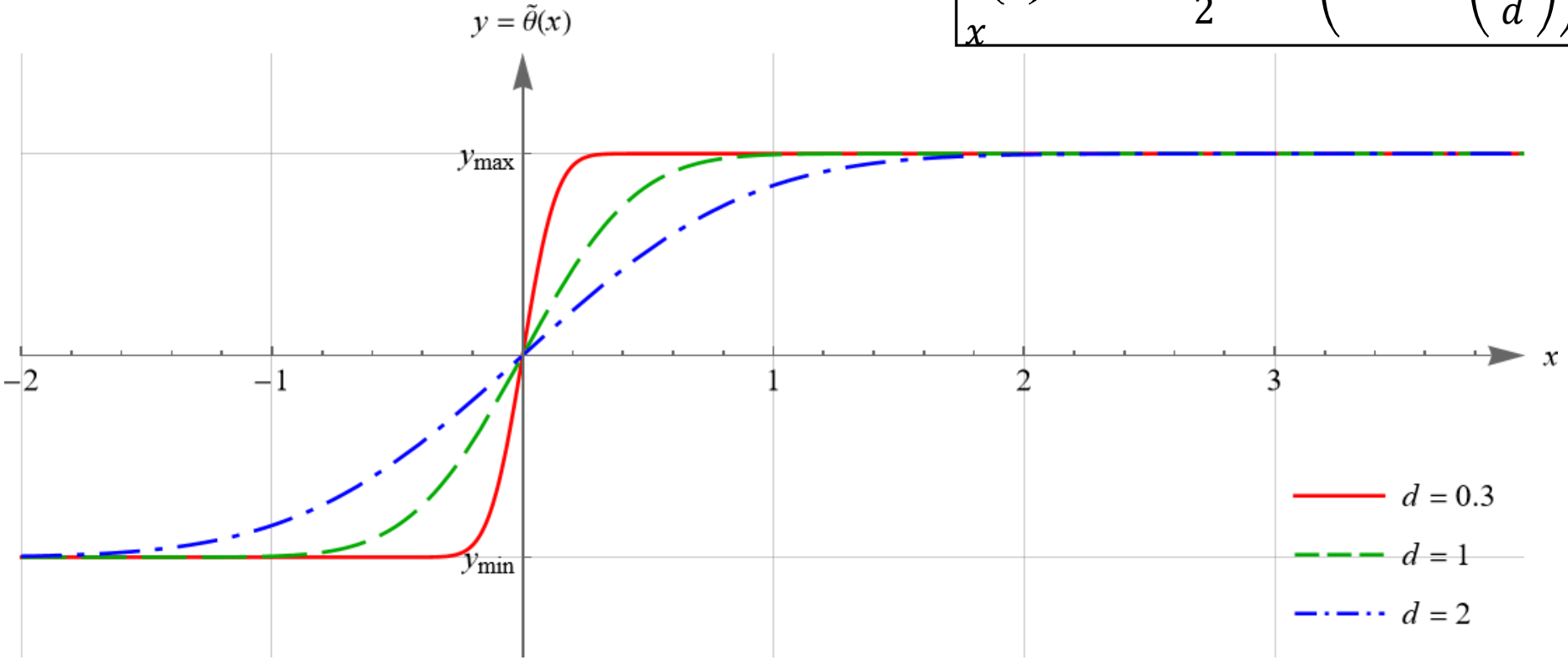


```
reg1d
= ToElementMesh[Cuboid[{0, 0, 0}, {a, b, L1d}], MaxCellMeasure
ElementMesh[{{0., 0.02286}, {0., 0.01016}, {0., 0.2}}, {HexahedronElement[⟨4000⟩]}]
```



Функция в виде гладкой ступеньки

$$\tilde{\theta}(x) = \frac{y_{max} - y_{min}}{2}\left(1 + \operatorname{erf}\left(\frac{2x}{d}\right)\right) + y_{min}$$

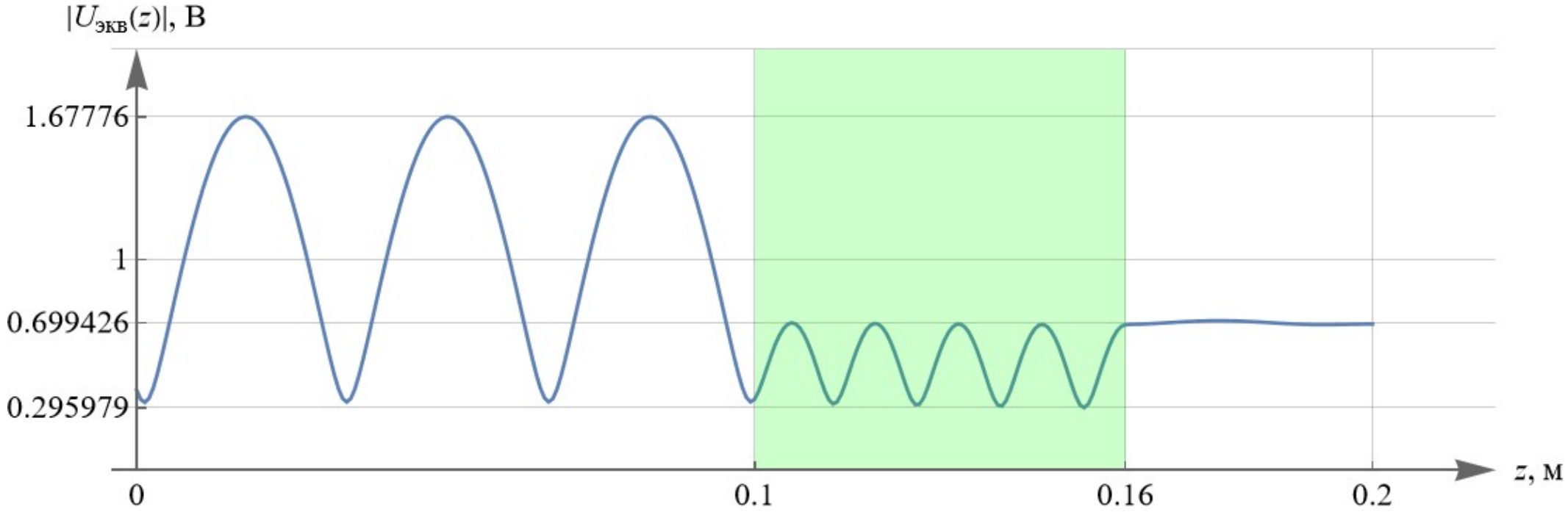


$$\operatorname{erf} x = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

$$\forall_{x \in \mathbb{R}} \operatorname{erf} x \in (-1; 1)$$

$$\text{SmoothStep}[x_, ymin_, ymax_, d_] := \frac{ymax - ymin}{2}\left(1 + \text{Erf}\left[\frac{2}{d} * x\right]\right) + ymin;$$

Режим смешанных волн



$$\text{КСВ}_{\text{д}} \approx \frac{0.699426}{0.295979} \approx 2.36 \Longrightarrow |\Gamma| = \frac{\text{КСВ}_{\text{д}} - 1}{\text{КСВ}_{\text{д}} + 1} \approx 0.41$$

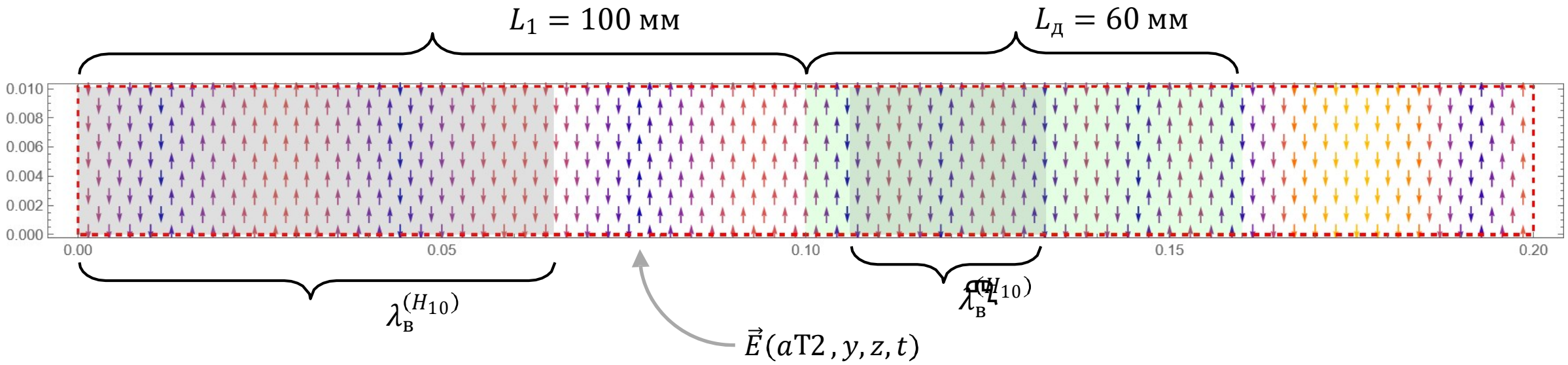
$$|\Gamma_{\text{кл}}| = \left|\frac{R - R_{\text{в}}}{R + R_{\text{в}}}\right| \overset{R \sim 1\text{Т}\beta}{=} \left|\frac{\beta - \beta_{\text{в}}}{\beta + \beta_{\text{в}}}\right| \overset{\beta \approx 2\pi\text{Т}\lambda_{\text{в}}}{=} \left|\frac{\lambda_{\text{д}}^{(H_{10})} - \lambda_{\text{в}}^{(H_{10})}}{\lambda_{\text{д}}^{(H_{10})} + \lambda_{\text{в}}^{(H_{10})}}\right| \approx \left|\frac{27.0 - 65.4}{27.0 + 65.4}\right| \approx 0.42 \blacksquare$$

Поле в волноводе с диэлектриком

$f = 8.0$ ГГц, $a = 22.86$ мм, $b = 10.16$ мм, $L = 200$ мм

$\varepsilon_{\text{в}} = 1.00059$, $\operatorname{tg} \mathrm{P}_{\text{в}} = 1 \cdot 10^{-15}$, $\mu_{\text{в}} = 1$, $\varepsilon_{\text{д}} = 2.6$, $\operatorname{tg} \mathrm{P}_{\text{д}} = 0.004$, $\mu_{\text{д}} = 1$

$$\lambda_{\text{в}}^{(H_{10})} = \frac{\lambda_{\text{c}}}{\sqrt{1 - \left(\lambda_{\text{c}}^{(H_{10})}/\lambda_{\text{кр}}^{(H_{10})}\right)^2}} \approx 27.0 \text{ мм}, \qquad \lambda_{\text{c}} = \frac{v_{\text{ф}}}{f} = \frac{1}{f\sqrt{\varepsilon_{\text{д}}\varepsilon_0\mu_{\text{д}}\mu_0}} \approx 23.2 \text{ мм}, \qquad \lambda_{\text{кр}}^{(H_{10})} = 2a \approx 45.7 \text{ мм}$$

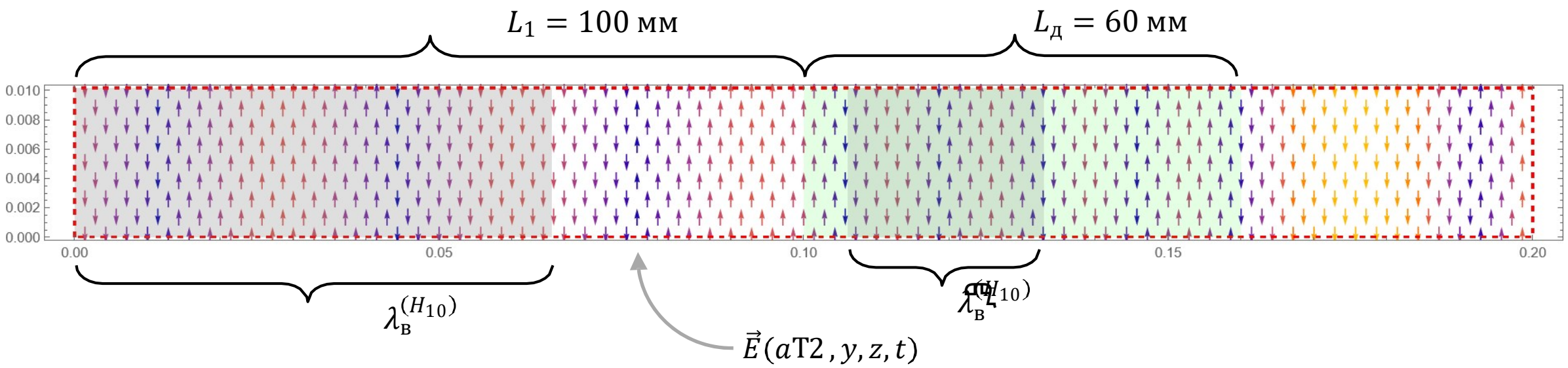


Процессы в волноводе с диэлектриком

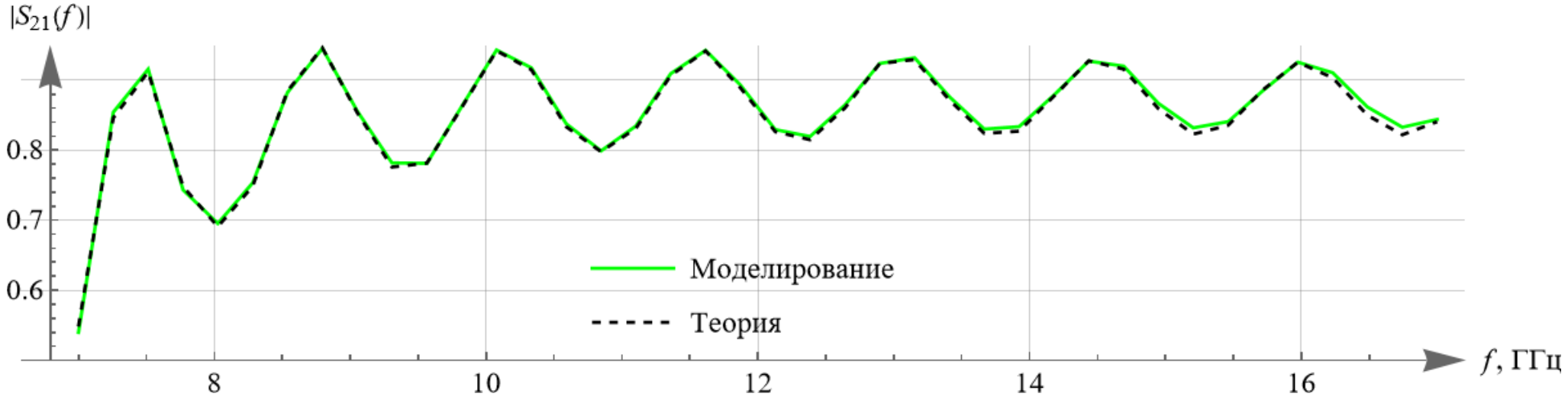
$f = 8.0$ ГГц, $a = 22.86$ мм, $b = 10.16$ мм, $L = 200$ мм

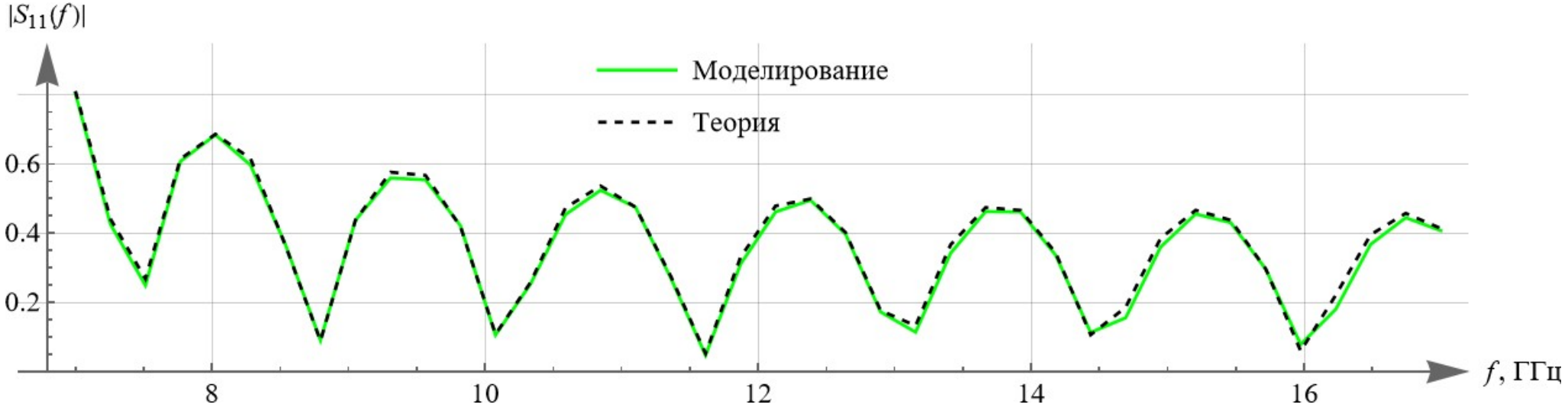
$\varepsilon_{\text{в}} = 1.00059$, $\operatorname{tg} \mathrm{P}_{\text{в}} = 1 \cdot 10^{-15}$, $\mu_{\text{в}} = 1$, $\varepsilon_{\text{д}} = 2.6$, $\operatorname{tg} \mathrm{P}_{\text{д}} = 0.004$, $\mu_{\text{д}} = 1$

$$\lambda_{\text{в}}^{(H_{10})} = \frac{\lambda_{\text{c}}}{\sqrt{1 - \left(\lambda_{\text{c}}^{(H_{10})}/\lambda_{\text{кр}}^{(H_{10})}\right)^2}} \approx 27.0 \text{ мм}, \qquad \lambda_{\text{c}} = \frac{v_{\text{ф}}}{f} = \frac{1}{f\sqrt{\varepsilon_{\text{д}}\varepsilon_0\mu_{\text{д}}\mu_0}} \approx 23.2 \text{ мм}, \qquad \lambda_{\text{кр}}^{(H_{10})} = 2a \approx 45.7 \text{ мм}$$

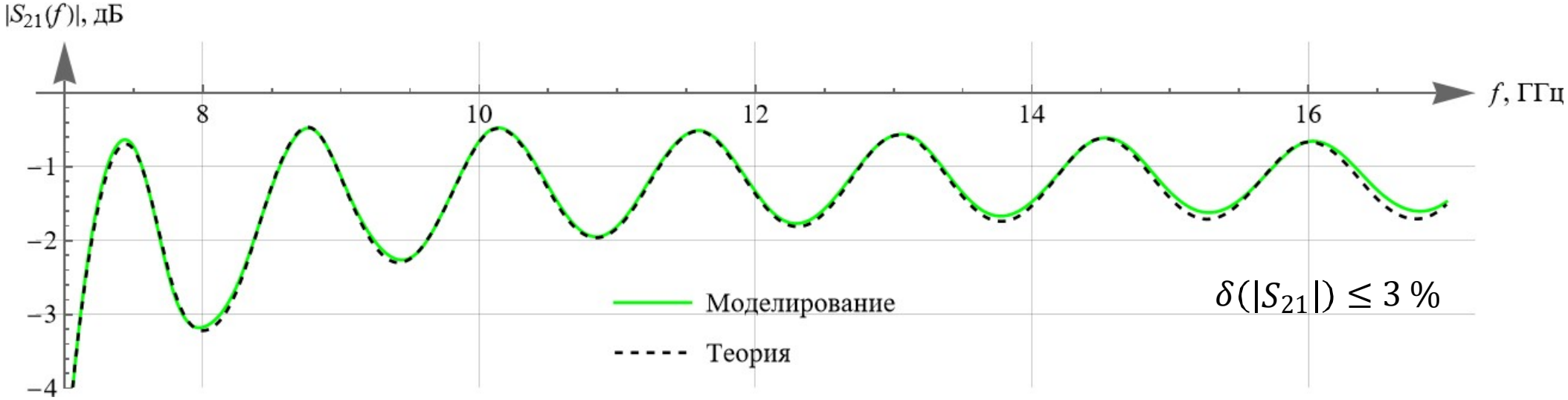


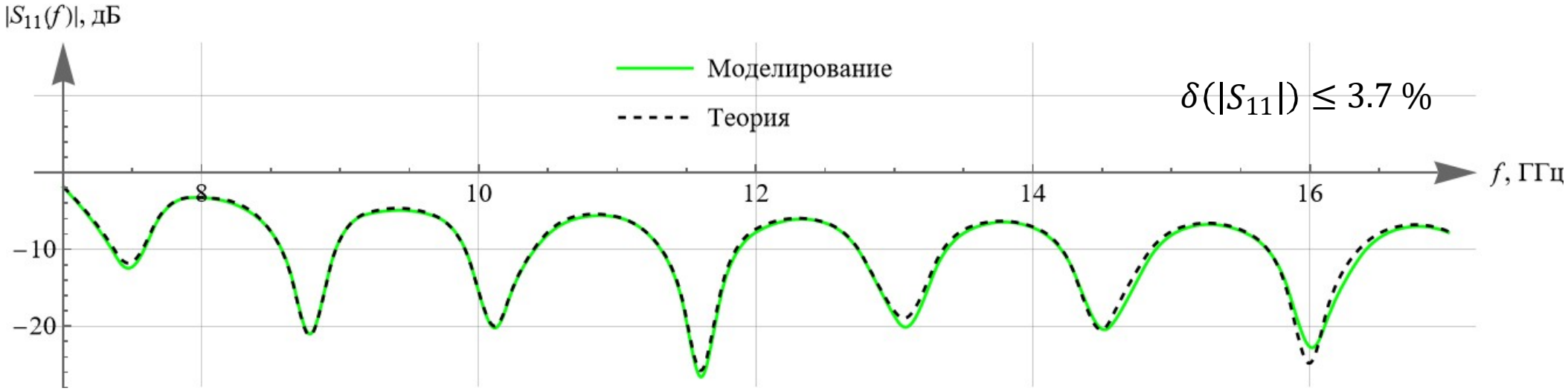
Амплитудные параметры рассеяния



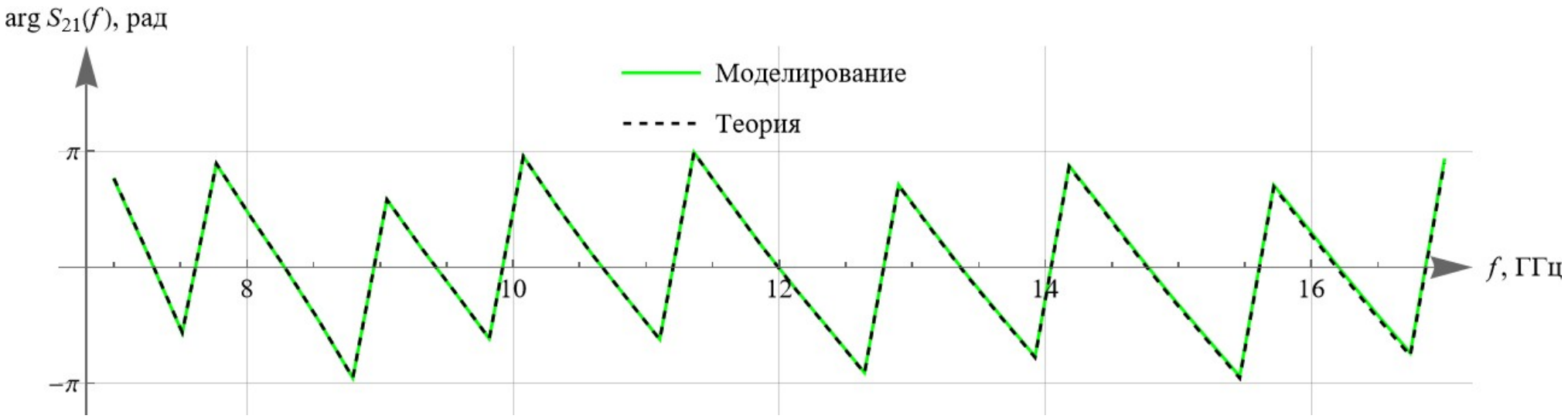


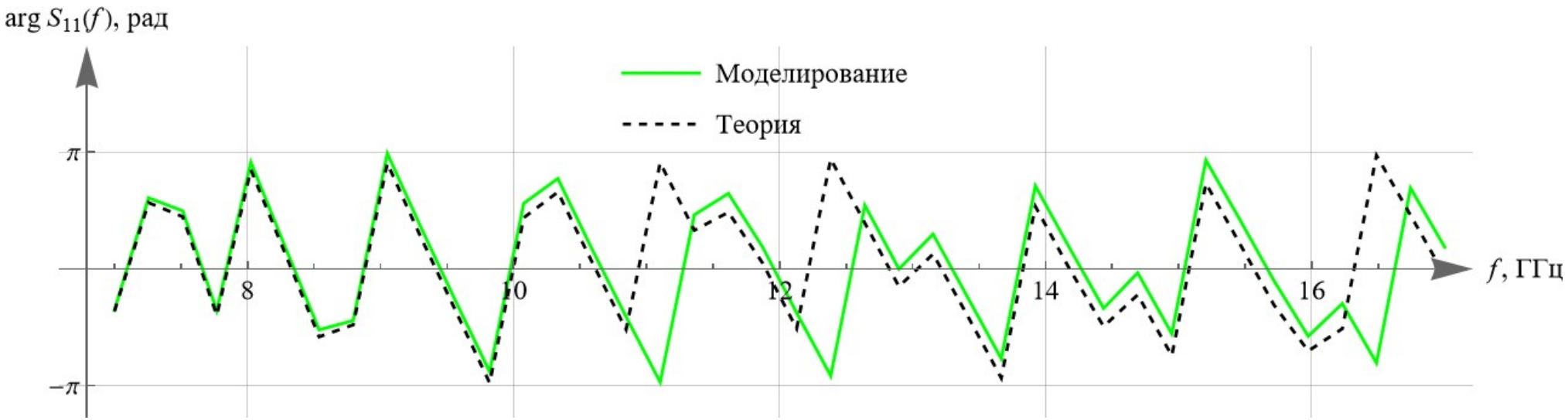
Интерполяция параметров рассеяния





Фазовые параметры рассеяния





Спасибо за внимание