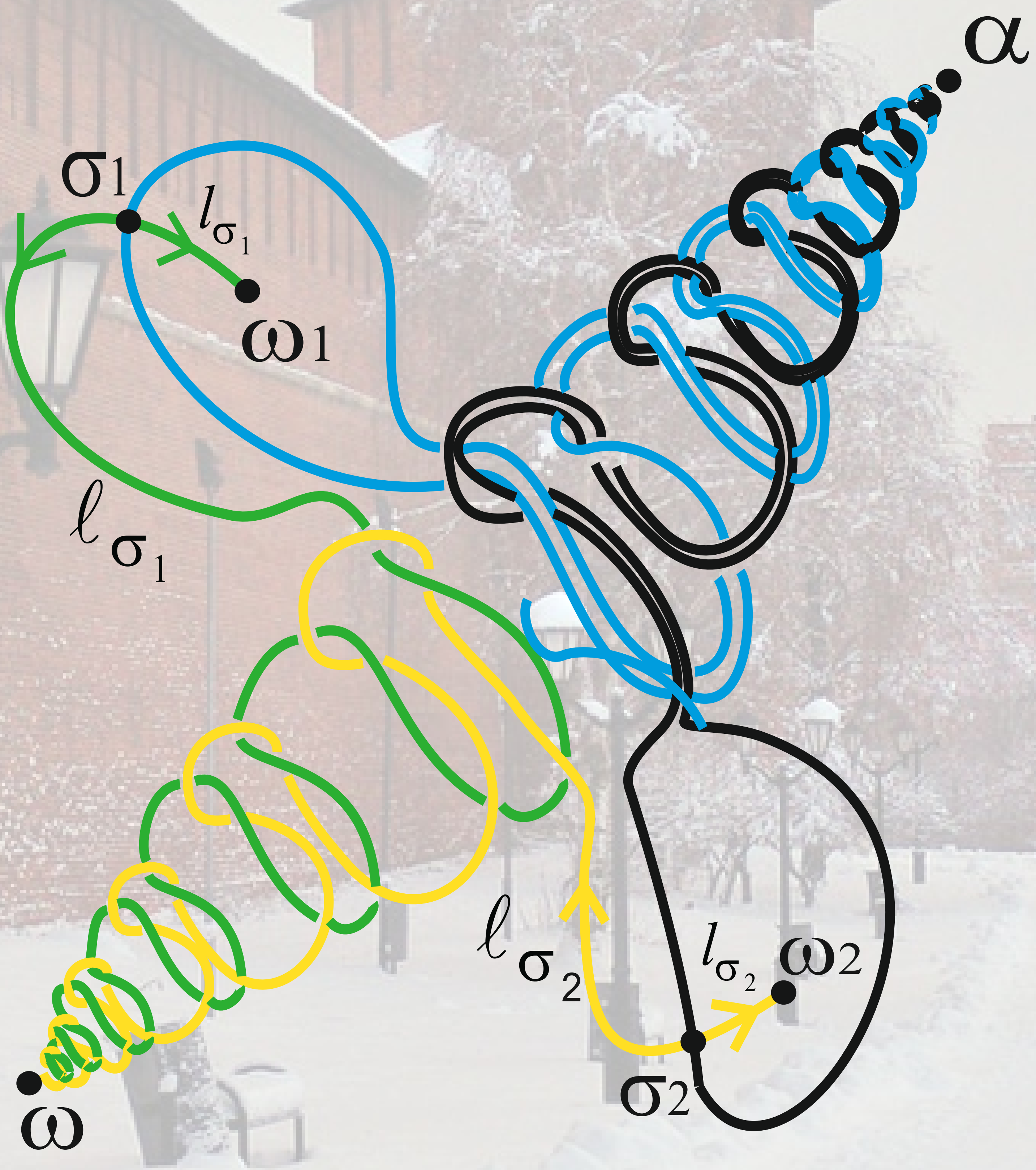
International Conference
«Topological methods in dynamics and related topics»

Book of Abstracts



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A Remark on the Mahowald's Elements in Stable Homotopy Groups of Spheres

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A geometrical description of the Mahowald elements [2] of dimensions 2^i, $i \geq 3$ is presented. The result is an elementary geometrical reformulation of [3], based on skew-framed immersion cobordism group [1].

This is a joint result with Th. Yu. Popelenskii and O.D. Frolkina.

References

[1] P. M. Akhmet'ev, O. D. Frolkina, "On properties of skew-framed immersions cobordism groups", Fundam. Prikl. Mat., 21:5 (2016), 19-46., http://arxiv.org/abs/1712.00959v1.

[2] M.Mahowald, *A new infinite family in* ${}_2\pi_*^S$, Topology vol. 16 (1977) 249-256.

[3] Peter J. Eccles, Hadi Zare, *The Hurewicz image of the η_i family, a polynomial subalgebra of* $H_*(\Omega_0^{2^{i+1}-8+k} S^{2^i-2})$ arXiv:0909.3791v1 [math.AT] 21 Sep 2009.

Convexity and its Applications in the Theory of Two-dimensional Dynamical Systems

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The report deals with the two-dimensional system of ordinary differential equations:

$$\dot{x} = f(x, y) \qquad \dot{y} = g(x, y) \tag{1}$$

possessing by periodic solution γ with period T, this plane closed curve γ being the boundary of some plane convex region K with the support function $p(\varphi)$: $\gamma \equiv \partial K$.

In this work we show that the assumption about convexity of K gives us the possibility to estimate a number of important values concerning the system (1) in the absence of information about the explicit form for exact solution of this system.

At first let us take into account the diameter D and the width Δ for the convex region K ($D \geq \Delta$):

$$D = \sup_{\varphi \in [0, 2\pi]} B(\varphi) \qquad \Delta = \inf_{\varphi \in [0, 2\pi]} B(\varphi) \tag{2}$$

where $B(\varphi) = p(\varphi) + p(\pi + \varphi)$ is the breadth of the convex region K in the direction φ.

The following inequalities for the length $L = \oint_\gamma ds$ ($ds = \sqrt{\dot{x}^2 + \dot{y}^2}\, dt$) of K and the surface area $F = \frac{1}{2} \oint_\gamma x\, dy - y\, dx$ of K including parameters (2) is known to hold [1] :

$$2\sqrt{D^2 - \Delta^2} + 2\,\Delta \arcsin\frac{\Delta}{D} \leq L \leq 2\sqrt{D^2 - \Delta^2} + 2\,D \arcsin\frac{\Delta}{D} \tag{3}$$

and

$$\Delta\, D \leq 2\, F \leq \Delta \sqrt{D^2 - \Delta^2} + D^2 \arcsin \frac{\Delta}{D} \,. \tag{4}$$

In particular let K be convex region with constant breadth then both of values (2) coincide and from inequality (3) one can immediately obtain that in this case $L = \pi\, D$.

It means that if $(x(t),\, y(t))$ is unknown solution of system (1) corresponding to ∂K of such shape then one can calculate exactly the next integral:

$$\int_0^T \sqrt{f^2(x(t), y(t)) + g^2(x(t), y(t))}\, dt = \pi\, D \,. \tag{5}$$

Further let us consider the Fenchel inequality [2] including curvature k of curve γ:

$$\oint_\gamma k\, ds \geq 2\, \pi \,. \tag{6}$$

If K is convex region then on unknown solution $(x(t),\, y(t))$ of system (1) defining γ inequality (6) is reduced to the following identity:

$$\int_0^T \frac{|\, \dot{x}\, \ddot{y} - \dot{y}\, \ddot{x}\, |}{\dot{x}^2 + \dot{y}^2}\, dt = 2\, \pi \,, \tag{7}$$

where derivatives of the second order one can extract from system (1):

$$\ddot{x} = f(x,y)\, \frac{\partial f(x,y)}{\partial x} + g(x,y)\, \frac{\partial f(x,y)}{\partial y} \qquad \ddot{y} = f(x,y)\, \frac{\partial g(x,y)}{\partial x} + g(x,y)\, \frac{\partial g(x,y)}{\partial y} \,.$$

In the report applications of formulae (3)-(5) and (7) to the Hamilton systems

$$H(x,y) = \frac{y^2}{2} + U(x) \tag{8}$$

with different potential energies $U(x)$ obeying to the requirement $U''(x) \geq 0$ will be discussed.

We underline that for system (8) there is a simple relation $F = 2\, \pi\, I$ between the surface area F and the action variable I corresponding to curve γ hence inequality (4) is very fruitful for studying of such systems because of parameters (2) can be easily calculated for a wide range of Hamiltonians (8).

The results described above may be useful under solution of the second part of Hilbert's 16th problem and the weakened Hilbert's 16th problem (see [3, 4] and references there in).

References

[1] Kubota T. Einige Ungleichheitsbeziehungen über Eilinien und Eiflächen // Sci. Rep. Tôhoku Univ. 1923. V.12. P. 45-65.

[2] Fenchel W. Über Krümmung und Winding geschlossener Raumkurven // Math. Ann. 1929. Bd. 10. S. 238-252.

[3] Glutsyuk A.A., Ilyashenko Yu.S. Restricted version of the infinitesimal Hilbert 16th problem // Mosc. Math. J. 2007. V. 7. Issue 2. P. 281–325.

[4] Morozov A.D. Quasi-conservative systems: cycles, resonances and chaos. – Singapore: World Sci, in ser. Nonlinear Science, ser. A, vol. 30, 1998. – 325 p.

Gromov-Hausdorff Hyperspaces of $\mathbb{R}^n$

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Let (M, d) be a metric space. For two non-empty subsets A, $B \subset M$, the Hausdorff distance $d_H(A, B)$ is defined as follows: $d_H(A, B) = \max\{\sup_{a\in A} d(a, B),\ \sup_{b\in B} d(b, A)\}$, where $d(x, C) = \inf\{d(x, c) \mid c \in C\}$. The set of all non-empty compact subsets of M is denoted by 2^M and is endowed with the Hausdorff metric d_H. The pair $(2^M, d_H)$ is called the hyperspace of M.

For two compact metric spaces X and Y, their Gromov-Hausdorff distance $d_{GH}(X, Y)$ is defined to be the infimum of all Hausdorff distances $d_H(i(X), j(Y))$ for all metric spaces M and all isometric embeddings $i : X \to M$ and $j : Y \to M$. It is a useful tool for studying topological properties of families of metric spaces.

Clearly, the Gromov-Hausdorff distance between two isometric spaces is zero; it is a metric on the family $\mathbb{GH}$ of isometry classes of compact metric spaces. The metric space $(\mathbb{GH}, \mathrm{d_{GH}})$ is called the Gromov-Hausdorff space.

In this talk we mainly are interested in the subspace $\mathbb{GH}(\mathbb{R}^n)$ of $\mathbb{GH}$ consisting of the classes $[E] \in\mathbb{GH}$ whose representative E is a metric subspace of the standard Euclidean space $\mathbb{R}^n$, $n \geq 1$. $\mathbb{GH}(\mathbb{R}^n)$ is called the Gromov-Hausdorff hyperspace of $\mathbb{R}^n$. One of the main results in this talk asserts that $\mathbb{GH}(\mathbb{R}^n)$ is homeomorphic to the orbit space $2^{\mathbb{R}^n}/E(n)$, where $2^{\mathbb{R}^n}$ is the hyperspace of all non-empty compact subsets of $\mathbb{R}^n$ endowed with the Hausdorff metric, and $E(n)$ is the isometry group of $\mathbb{R}^n$. This is applied to describe the topological type of $\mathbb{GH}(\mathbb{R}^n)$.

How Many Ways are there to Tile a Rectangle with Polyominoes?

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Abstract. We discuss the statistical weight problem for a system of graphs embedded in a square lattice with respect to interrelations with the cluster characterizing problem. On simple examples we explore difficulties arising in generating function constructions and analyze possibilities of techniques generalization.

Introduction. A polyomino is a plane geometric figure formed by joining equal squares. An n-omino is a set of n rookwise connected unit squares [1].

Finding a graph covering of a lattice is equivalent to tiling an area with polyominoes. This problem reduces to building a generating function $G(z)$. The following techniques to obtain the tilings are introduced and investigated.

Direct method of counting the number of partitions. Suppose a $m \times n$ stripe and a set of polyominoes are given. Construct all the possible partitions of the stripe, where a monomino, that is a 1×1 square, is denoted by z, graphically. Generating functions for domino tilings are $G(z) = \frac{1}{1-z^2}$ for a $1 \times n$ stripe, $G(z) = \frac{1}{1-z^2-z^4}$ for a $2 \times n$ stripe, $G(z) = \frac{1-z^6}{1-4z^4+z^{12}}$ for a $3 \times n$ stripe.

Indirect method of counting the number of partitions. Supposed there is a finite graph G given, a solution of the domino tiling problem is equivalent to a dimer arrangement which contains all points of G [2]. Introduce the multiplication operation for dimer partitions of a $2 \times n$ stripe and direct vertical dimers to avoid identical results from different combinations. Square roots of the following function‘s formal power series expansion coefficients give the number of possible tilings: $G(z) = \frac{1}{1-(2z+z^2)+z^3}$.

Whereas the solution to the dimer tiling problem of a $2 \times n$ field is already known, the use of this method could lead to solving more general problems.

Coloured digraphs method. Suppose there are a rectangular $m \times n$ field and a set of arbitrary tiles. Then the coloured digraphs method can be applied to obtain all the possible partitions of the given field.

The graph assembly is represented by coloured oriented trees, where adjacent vertices may be of the same colour. The operations applied to the field units, 1×1 squares, are denoted by colours of vertices. There are four operations of joining a square: without forming a spring, with forming a horizontal spring, with forming a vertical spring and with forming both horizontal and vertical springs, and the operation of translation. Then the number of all possible partitions of a $m \times n$ field into arbitrary tiles is $2^{2mn-m-n}$.

Suggested applications of the digraph method. Consider the following questions connected with the problem of structural complexity of a cluster for the development of the method.

- How many ways are there to form a cluster with a given boundary from primitives of a fixed number and types?
- Derive the structural complexity of an arbitrary cluster from the complexity of its assembly and the complexity of generating the primitive class.
- Consider enumerating the structures of primitives for a fixed rectangular field and arbitrary numbers of primitives.
- Enumerate the ways to assemble certain primitives and choose the optimal way for the given structure of joining and field size.

References

[1] Solomon W. Golomb. Polyominoes Which Tile Rectangles. Journal of Combinatorial Theory, Series A 51, 117-124, 1989.

[2] Kasteleyn, P. W. (1967), "Graph theory and crystal physics", in Harary, F., Graph Theory and Theoretical Physics, New York: Academic Press, pp 43–110.

On Topologycal Properties of the Volume Entropy of Geodesic Flows

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In 1979 Mannig estimated from bellow the topological entropy of the geodesic flow of a Riemannian metric on some manifold by the growth rate of the geodesic balls on its universal covering. Nowadays this growth rate is known as asymptotic volume or *volume entropy* of the geodesic flow. If we change the given metric in the class of metrics of constant volume equal to 1 the infimum of the volume entropy, if it is positive, provides an interesting topological invariant of the given manifold.

In the talk we give a little review of old known results in the direction as well as we establish the recent development.

Lorenz-type Attractor in a Piecewise Smooth System: Rigorous Results

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In this talk we consider a glued 3D system A composed from linear subsystems A_s,A_l and A_r, $(x, y, z) \in R^3$. The subsystems A_s,A_l and A_r are defined in the domains G_s,G_l and G_r respectively, where

$$\begin{array}{l} G_s : |x| < 1, y \in R^1, z < b, \\ G_l : (x \leq -1, z \leq b) \cup (x < -\mathrm{sign} y, z > b), y \in R^1, \\ G_r : (x \geq 1, z \leq b) \cup (x > -\mathrm{sign} y, z > b), y \in R^1, \end{array} \tag{1}$$

b is a positive parameter. Subsystems are defined as follows

$$\begin{array}{rcl} A_s & : & \dot{x} = x,\ \dot{y} = -\alpha y,\ \dot{z} = -\nu z, \\ & & \\ & & \dot{x} = -\lambda(x \pm 1) \pm \omega(z - b), \\ A_{l,r} & : & \dot{y} = -\beta(y \pm 1), \\ & & \dot{z} = \mp\omega(x \pm 1) - \lambda(z - b), \end{array} \tag{2}$$

where $\alpha > 0$, $\beta > 0$ and ν, ω, λ are positive parameters.

For the system (1),(2), we analytically obtain the Poincare return map in explicit form. This allows us to rigorously get all principle bifurcations of the system, in particular, the bifurcations of the birth and death of a strange attractor. Analytically obtaining expressions for Lyapunov exponents on trajectories of the attractor, we proved that this attractor is singularly hyperbolic, i.e. "genuinely" strange. The corresponding theorems are given.

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Periodic and Bounded Solutions, and Bifurcations of Functional-differential Equations of Pointwise Type

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We consider a functional differential equation of pointwise type (FDEPT)

$$\dot{x}(t) = f(t, x(q_1(t), \ldots, x(q_s(t))), \quad t \in B_R, \tag{1}$$

where $f : \mathbb{R} \times \mathbb{R}^{ns} \longrightarrow \mathbb{R}^n$ — mapping of the $C^{(0)}$ class; $q_j(\cdot), j = 1, \ldots, s$ — homeomorphisms of the line preserving orientation; B_R is either closed interval $[t_0, t_1]$ or closed half-line $[t_0, +\infty[$ or line $\mathbb{R}$.

The approach proposed for the study of such equations is based on a formalism whose central element is the construction using a finitely generated group

$$Q =< q_1, \ldots, q_s >$$

of homeomorphisms of the line (the group operation in such a group is the superposition of two homeomorphisms).

The importance of equations of the considered type is determined by the fact that the theory of solutions of such equations is closely related to the theory of soliton solutions for infinite-dimensional ordinary differential equations [1]. The equation (1) defined on the entire line canonically induce an infinite-dimensional ordinary differential equation. In this case, to each solution of the equation (1) there corresponds a one-to-one solution of the traveling wave type of the induced infinite-dimensional equation. In particular, finite-difference analogs of equations of mathematical physics define infinite-dimensional dynamical systems. Important subclasses of solutions of the traveling wave type of infinite-dimensional ordinary differential equations are periodic and bounded solutions of the traveling wave type, to which the periodic and bounded solutions of the induced functional differential equations correspond.

The main goal in the study of such differential equations is the investigation of the initial-boundary value problem

$$\dot{x}(t) = f(t, x(q_1(t), \ldots, x(q_s(t))), \quad t \in B_R, \tag{2}$$

$$\dot{x}(t) = \varphi(t), \quad t \in \mathbb{R} \backslash B_R, \quad \varphi(\cdot) \in L_\infty(\mathbb{R}, \mathbb{R}^n), \tag{3}$$

$$x(\bar{t}) = \bar{x}, \quad \bar{t} \in \mathbb{R}, \quad \bar{x} \in \mathbb{R}^n, \tag{4}$$

which we will call the *basic initial-boundary value problem*. In a general situation, when $\bar{t} \neq t_0, t_1$, or deviations of the argument are arbitrary, we have a problem with *non-local initial-boundary conditions*. If the group Q is trivial, the initial-boundary problem 2-4 goes into the well-known Cauchy problem for ordinary differential equations (ODE).

Solutions of the initial-boundary value problem under consideration do not inherit such remarkable properties of ODEs as the existence and uniqueness of the solution, the pointwise completeness of the solution space, the n-parametry of the solution space, the stability of the equation, etc. However, within the framework of the proposed formalism, it is possible to describe the procedures for expanding the class of ODEs while preserving one or another property. In particular, an extension procedure is described, in which there are no branching solutions, as well as a violation of point completeness.

References

[1] *Beklaryan L.A.* Introduction to the theory of functional differential equations. Group approach. Factorial Press, Moscow, 2007. (in Russian)

[2] *Beklaryan L.A.* A method for the regularization of boundary value problems for differential equations with deviating argument // Soviet Math. Dokl. 1991. 43. 567–571.

[3] *Beklaryan L.A.* A new approach to the issue of the existence of periodic solutions for functional differential equations of pointwise type // Izvestiya: Mathematics. 2018. 82. (in print)

Evolution and Controlling of Hyperbolic the Plykin - Newhouse Attractor by the Pyragas Method

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In the present contribution consideration is being given to an autonomous physical system which is characterized by the presence of the attractor of a hyperbolic type. We study the possibility of controlling and evolution the Plykin - Newhous attractor of this type by the Pyragas method. The choice of the method of control. As such it is possible to use an external signal or the introduction of additional delayed feedback.

Introduction. Hyperbolicity is a fundamental feature of chaotic systems. It is as follows: a tangent space Σ of such systems is a combination of three subspaces; stable E^s, unstable E^u and E^0 neutral. Close trajectories which correspond to converge exponentially to E^s each other when $t \to +\infty$, and those which correspond to E^u - when $t \to -\infty$. In the subspace E^0 the vectors contract and expand more slowly than the exponential velocity. When the degree of contraction and expansion in the subspaces E^s and E^u changes from point to point along the trajectory, such systems are called non-uniformly hyperbolic. Dynamic systems with uniform hyperbolicity of all the trajectories are called Anosov systems. Smale - Williams' solenoid and Plykin's attractor [1] are well- known hyperbolic attractors. Plykin's sphere is obtained by the transformation of the disc domain into itself where S^2 - a unitary disc in R^2. Then $f : T \mapsto \mathbb{T}, f(x, y, z) = (\cos\varphi \sin\phi, \sin\varphi \sin\phi, \cos\phi)$, where $k>2$ determines the compression "by thickness", sets the disc as a subset $\mathbb{T} \subset \mathbb{R}^3$. Let there be a smooth family of non - linear controlled systems of ordinary differential equations $\dot{x} = F(x, \mu, u), x \in M \subset R^m, \mu \in L \subset R^k, u \in U \subset R^n, F \in C^\infty$ depending on the vector of controlling parameters u. Suppose that it is necessary to stabilize unstable limiting cycle of the period T, which is the solution of the family when $u = 0$. Let the system have a regular attractor when the parameters are of the same value $u = 0$. Then the stabilization of the cycle is carried out by means of the feedback with the delay being in the form of $u(t) = K(x(t-T) - x(t))$, where K - is the matrix of coefficients [2].

The use of the Pyragas method for the formation of regular dynamics in autonomous hyperbolic attractors. Let us take into consideration the system of the type [1]:

$$\begin{cases} \dot{X} = -2\epsilon Y^2 \Omega_1 (\cos(\omega_2 \cos\omega_1 t) - X \sin(\omega_2 \cos\omega_1 t)) + \\ \qquad kY\Omega_2(\cos(\omega_2 \sin\omega_1 t) - X\sin(\omega_2 \sin\omega_1 t)) \sin\omega_1 t, \\ \dot{Y} = 2Y\Omega_1(X\cos(\omega_2\cos\omega_1 t) + 2^{-1}(1 - X^2 + Y^2)\sin(\omega_2\cos\omega_1 t)) - \\ \qquad k\Omega_2(X\cos(\omega_2\sin\omega_1 t) + 2^{-1}(1 - X^2 + Y^2)\sin(\omega_2\sin\omega_1 t))\sin\omega_1 t + F(K, \tau), \\ \Omega_1 = (2X\cos(\omega_2\cos\omega_1 t) + (1 - X^2 - Y^2)\sin(\omega_2\cos\omega_1 t))(1 + X^2 + Y^2)^{-2}, \\ \Omega_2 = (-2X\sin(\omega_2\sin\omega_1 t) + (1 - X^2 - Y^2)\cos(\omega_2\sin\omega_1 t))(1 + X^2 + Y^2)^{-1} + 2^{-1/2}. \end{cases}$$

Here X, Y - dynamic variables, ϵ and k - coefficient of connection, $\omega_{1,2} = (\pi/2, \pi/4)$ -inherent frequency oscillations, $F_{Y,\tau} = K[Y(t-\tau) - Y(t)]$.

Result. The $K = 0$ hyperbolic chaotic state, corresponds to $K = 1.8$ and $\tau = 1.8$ the stable state, corresponds to the chaotic state.

References

[1] S. Kuznetsov. An example of a non-autonomous continuous-time system with attractor of Plykin type in the Poincare map. *Nonlinear dynamics* **5**: 403–424, 2009.

[2] K. Pyragas. Continuous control of chaos by self-controlling feedback.*Physics Letters A* **170**: 421–428, 1992.

Emergence of Non-ergodic Dynamics Representations of Simple Compact Connected Lie Groups

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Recently wild dynamics were shown locally typical of in the sense of Kolmogorov. These dynamics are wild in the sense that they seem difficult to approximate by a finitely ergodic system. In order to quantify this complexity, I introduced the emergence $\mathcal{E}(\epsilon)$ as the minimal number N of probability measures $(\mu_i)_{1\le i\le N}$ such that the empirical function $x \mapsto E_k(x) := \frac{1}{k}\sum_{i=1}^{k} \delta_{f^i(x)}$ satisfies:

$$\limsup_{k\to\infty} \int_M d_{W_1}(E_k(x), \{\mu_i : 1 \le i \le N\})dLeb < \epsilon \ ,$$

where d_{W_1} is the 1-Wasserstein metric on the space of probability measures.

I will present a program which aims to 1. Show the typicality of dynamics with high emergence, 2. Describe dynamics with high emergence using a dictionary with the notion of entropy.

Such a program will be illustrated by recent achievements in several collaborations.

Partially Hyperbolic Diffeomorpihsms on 3-manifolds

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A diffeomorphism f of a closed 3 manifold M is partially hyperbolic if the tangent bundle TM splits in three 1-dimensional Df-invariant bundle, $TM = E^s + E^c + E^u$, the bundle E^s is uniformly contracted, E^u is uniformly expanded, and E^c is dominated by E^u and dominates E^s. The classical examples are

- the skew product of an Anosov diffeomorphism of the torus $\mathbb{T}^2$ by circle diffeomorphisms,
- perturbations of a linear Anosov diffeomorphism of $\mathbb{T}^3$ with 3 real eigenvalues of distinct moduli
- the perturbations of the time one map of an Anosov flow.

In 2000 E. Pujals proposed informally that these three classes of examples could be the general behavior of partially hyperbolic diffeomorphisms, leading to was has been known as Pujals conjecture. Many recent new examples disprove this conjecture, providing many unexpected behaviors. However with Jinhua Zhang we recently proved the conjecture under the assumption that the center bundle E^c is topologically neutral: a small segment tangent to E^c has all its iterates remaining small.

In this talk I will give a quick survey of the recent examples, and present some of the arguments of our proof.

Interpolation for Determinantal Point Processes

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Consider a Gaussian Analytic Function on the disk, that is, a random series whose coefficients are independent complex Gaussians. In joint work with Yanqi Qiu and Alexander Shamov, we show that the zero set of a Gaussian Analytic Function is a uniqueness set for the Bergman space on the disk: in other words, almost surely, there does not exist a nonzero square-integrable holomorphic function having these zeros. The key role in our argument is played by the determinantal structure of the zeros, and we prove, in general, that the family of reproducing kernels along a realization of a determinantal point process generates the whole ambient Hilbert space, thus settling a conjecture of Lyons and Peres. In a sequel paper, joint with Yanqi Qiu, we study how to recover a holomorphic function from its values on our set. The talk is based on the preprints arXiv:1806.02306 and arXiv:1612.06751

Controllability of Dynamic Inequalities and its Stability

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The concept of rough dynamical systems was introduced by A.A. Andronov and L.S. Pontryagin [1]. They analyzed family of phase curves of a C^1-vector field on a two-dimensional disk when the field does not vanish at the disk boundary and has no tangency with it and found the necessary and sufficient conditions such that for a field satisfying these conditions and any vector field sufficiently C^1 -close to it the families of phase curves of these fields are translated one into another by a homeomorphism being close to the identity. Later on M.Peixoto proved that rough vector fields are generic on any closed orientable surface [2, 3].

An analogous problem for dynamic inequalities was formulated in [4]. Such a problem naturally includes analysis of local controllability properties. Structural stability of generic control systems on closed orientable surfaces was proved in [5] (see also [6]), and for generic dynamic inequality this problem is open up to now, while the stability of local controllability properties of generic dynamic inequality with locally bounded derivatives is already proved [7] and structural stability of generic simplest dynamic inequality on S^2 is also proved [8].

We discuss these results and some related ones in other areas of mathematics.

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References

[1] *Andronov, A., Pontryagin, L.*, Rough systems//Dokl. Akad. Nauk SSSR 14(5), 247–250 (1937) (in Russian)

[2] *Peixoto M.M.* On Structural Stability//Annals of Mathematics, Second Series, Vol. 69, No. 1 (1959), pp. 199-222.

[3] *Peixoto M.M.* Structural stability on two-dimensional manifolds, Topology Volume 1, Issue 2, April–June 1962, pp. 101-120

[4] *Myshkis A.D.* On dynamic inequalities with locally bounded derivatives// Zap. mekh-mat dept and Kharkiv mathematical society, 1964. V.30. 152-163 (In Russian)

[5] *Davydov A.A.,* Structural stability of control systems on orientable surfaces//Math. USSR-Sb., 72:1 (1992), 1–28

[6] *Davydov A.A.* Qualitative theory of control systems. Translations of Mathematical Monographs, 141, American Mathematical Society, Providence, RI, 1994 , viii+147 pp.

[7] *Davydov A.A.,* Local controllability of typical dynamical inequalities on surfaces//Proc. Steklov Inst. Math., 209 (1995), 73–106

[8] *Grishina Yu.A., Davydov A.A.*, Structural Stability of Simplest Dynamical Inequalities//Proc. Steklov Inst. Math., 256 (2007), 80–91.

Phase Space Topology and Emergence of Chaos in First-order Mean Motion Resonances

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We consider a circular restricted tree body-problem – a system consisting of a star, a planet, and an asteroid near a first-order mean motion resonance. It means, that orbital periods of planet and asteroid are in relation $T_a : T_p \approx n : (n \pm 1)$. Long-term dynamics of the asteroid in this case is characterized by two integrals of motion: Hamiltonian value h and parameter $\sigma^2 = 1-(1-e^2)\cos^2 i$ arising from the rotational symmetry of the problem (e and i are eccentricity and inclination). We concentrate our attention on the region $e \ll \sigma \ll 1$ and show that from this part of the phase space the chaos in the system emerges. Dynamics in considered case is described by Hamiltonian, which has a universal form for all n:

$$H\left(\varphi, \Phi; x, \varepsilon^{-1}y\right) = \frac{\Phi^2}{2} + x\cos\varphi + y\sin\varphi + \cos 2\varphi. \tag{1}$$

Here (φ, Φ) and $\left(x, \varepsilon^{-1}y\right)$ are pairs of canonical variables, $x \approx \alpha_n e\cos\omega$, $y \approx -\alpha_n e\sin\omega$, ε – small parameter, α_n – constant coefficient, and ω – argument of pericenter.

Pairs of variables (φ, Φ) and (x, y) form two subsystems – fast and slow respectively – with significant difference in characteristic timescales due to ε in canonical counterpart of y. This allows to construct phase portraits of slow subsystem using averaging method. States corresponding to separatrices and stationary points of fast subsystem form a curve Γ on the slow subsystem's phase plane. The topological structure of slow phase portraits can be described by intersections of Γ with boundaries between different trajectory families (Fig. 1). Then bifurcations in phase portraits are revealed using h as a parameter (Fig. 1). General structure of phase portraits and bifurcations h_1-h_4, h_8 are preserved without the condition $\sigma \ll 1$ for all resonances except $1:2$.

Crossing of Γ by a trajectory cause a small quasi-random jump of a phase point [1]. Multiple passages through Γ thus lead to the so-called *adiabatic chaos*: all trajectories crossing Γ mix with each other and form a chaotic region (Fig. 2) with a characteristic time of diffusion ε^{-3} [1].

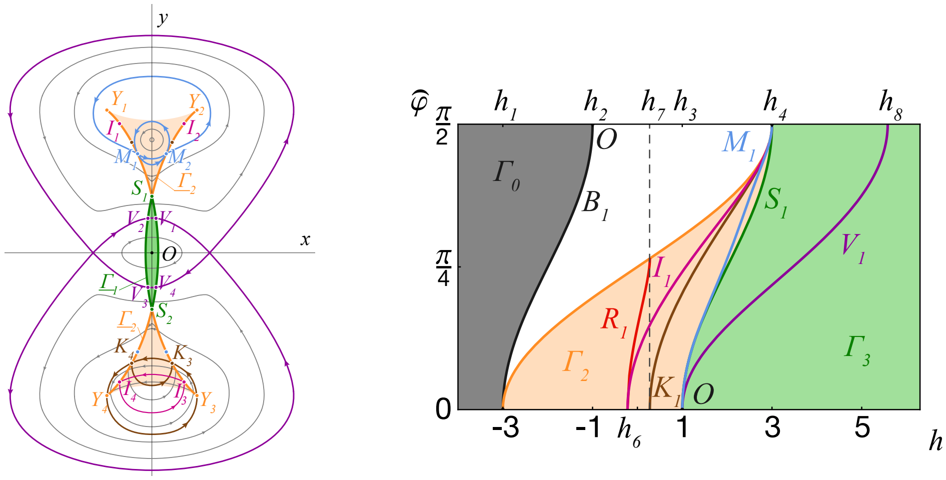

Figure 1: Slow phase portrait and intersection points V, M, I, K of Γ with boundary trajectories (left), bifurcation diagram showing positions of intersection points (right). Here $\widehat{\varphi}$ is parametrization of Γ: $x = \cos\widehat{\varphi}(h + \cos 2\widehat{\varphi} - 2)$, $y = \sin\widehat{\varphi}(h + \cos 2\widehat{\varphi} + 2)$

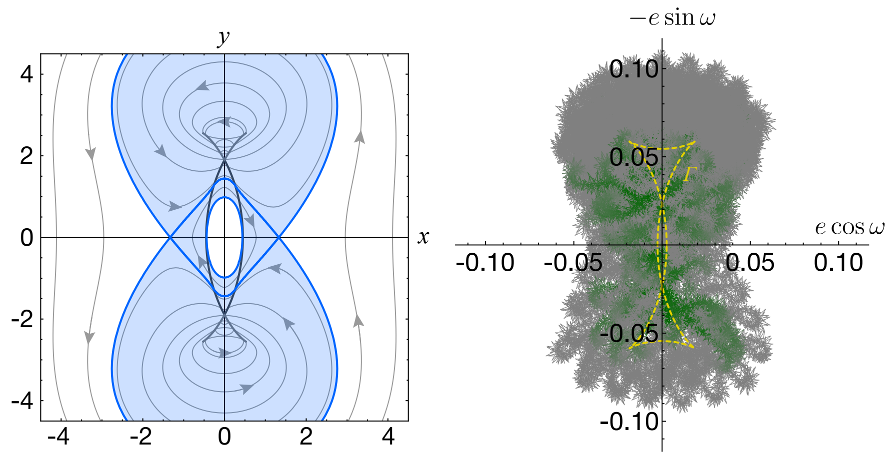

Figure 2: Chaotic region on the plane of slow variables (left). Object $1999CY_{13}$ from the outer Solar System filling the chaotic region during 15 Myr of evolution (right)

References

[1] Neishtadt, A.I.: Probability phenomena due to separatrix crossing. Chaos, **1**, 42, 42-48 (1991)

Ω-Stability of C^1-Smooth Skew Products of Interval Maps and Dense Stability as a Whole of a Family of Fibers Maps

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We give the survey of properties of C^1-smooth Ω-stable skew products of maps of an interval (with respect to homeomorphisms of skew products class) [1] – [3].

First of all, we describe the proper subspace of the space of C^1-smooth skew products of maps of an interval which contains the set of C^1-smooth Ω-stable skew products.

Then, we prove nondensity of the set of C^1-smooth Ω-stable skew products in the mentioned above subspace.

After that, we study the boundary of the set of C^1-smooth Ω-stable skew products in the space of all C^1-smooth skew products of maps of an interval.

In particular, we consider the set of C^1-smooth skew products of maps of an interval with densely stable as a whole (in C^1-topology) family of fibers maps and prove the criterion of C^1-approximability of maps with densely stable as a whole family of fibers maps with use of Ω-stable skew products of maps of an interval.

References

[1] *Efremova L.S.* Dynamics of skew products of interval maps // Russian Math. Surveys. 2017. 72, 1. 101–178.

[2] *Efremova L.S.* Ω-stable skew products of interval maps are not dense in $T^1(I)$ // Proc. Steklov Inst. Math. 2002. 236. 157-163.

[3] *Efremova L.S.* Stability as a whole of a family of fibers maps and Ω-stability of C^1-smooth skew products of maps of an interval // J. Phys.: Conf. Ser. 2016. 692. 012010.

Cantor Sets with High-dimensional Projections

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L. Antoine constructed a Cantor set in R^2 whose projections coincide with those of a regular hexagon [2, **9**, p.272; and fig.2 on p.273]. By K. Borsuk [5], there exists a Cantor set in R^d such that its projection onto every hyperplane contains a $(d-1)$-dimensional ball, or equivalently, has dimension $(d-1)$. J. Cobb [6] gives an example of a Cantor set in R^3 such that its projection onto every 2-plane is 1-dimensional (for higher-dimensional extensions, see e.g. [7], [3]). Cobb's method is rather sophisticated; the resulting Cantor set (as also the sets from [3], [7]) is tame in the following sense.

Definition 1. *A zero-dimensional compact set $K \subset R^n$ is called tame if there exists a homeomorphism h of R^n onto itself such that $h(K)$ is a subset of a straight line in R^n; and it is called wild otherwise.*

In R^2 each zero-dimensional compactum is tame [1, **75**, p. 87–89]. L. Antoine constructed a Cantor set in R^3 which is now widely known as Antoine's necklace [1, **78**, p. 91–92] and proved that it is wild [1, Part 2, Chap. III]. Similar construction gives uncountably many inequivalent Cantor sets in R^3 [9]; they all are called Antoine's necklaces.

Next two theorems provide a partial answer to the Cobb's question [6, p.126].

Theorem 1. *There exists an Antoine's Necklace A in R^3 such that for each plane $\Pi \subset R^3$, the projection $\dim p_\Pi(A)$ is a one-dimensional connected compactum.*

(Here $p_\Pi : R^3 \to \Pi$ is the orthogonal projection onto a plane $\Pi \subset R^3$.)

Next result implies that no projection of an Antoine's necklace can be (covered by) a set homeomorphic to a finite graph.

Definition 2. *A subset $X \subset R^N$ homeomorphic to a k-cube I^k is called flat if there exists a homeomorphism h of R^N onto itself such that $h(X)$ is a k-simplex.*

Theorem 2. *Let $X \subset R^n$ be a Cantor set. Suppose that for some m-dimensional plane Π, there exists a countable family of subsets $Y_1, Y_2, \ldots \subset \Pi$ such that each Y_i is a flat cell in Π, $\dim Y_i \leq m-1$, and $p_\Pi(X) \subset Y_1 \cup Y_2 \cup \ldots$. Then X is tame in R^n.*

In fact, each Cantor set in R^3 can be modified so that all its projections are 1-dimensional.

Theorem 3. *Let $X \subset R^3$ be any Cantor set. For each $\varepsilon > 0$ there exists a homeomorphism $h : R^3 \cong R^3$ such that $d(h, id) < \varepsilon$ and for each plane $\Pi \subset R^3$, we have $\dim p_\Pi(h(X)) = 1$.*

Finally, we find a new tameness condition which improves [4, Thm. 3E] (compare also with [8, Cor. 2]):

Theorem 4. *Let $X \subset R^n$ be a Cantor set. Suppose that there exists a plane Π such that $\dim \Pi \in \{1, 2, n-2, n-1\}$ and $\dim p_\Pi(X) = 0$. Then X is tame.*

References

[1] L. Antoine, Sur l'homéomorphie de deux figures et de leurs voisinages. Thèses de l'entre-deux-guerres, vol. 28, 1921. http://eudml.org/doc/192716

[2] L. Antoine, Sur les voisinages de deux figures homéomorphes, Fund. Math. 5 (1924) 265–287.

[3] S. Barov, J.J. Dijkstra, M. van der Meer, On Cantor sets with shadows of prescribed dimension, Topol. Appl. 159 (2012) 2736–2742.

[4] W.A. Blankinship, Generalization of a construction of Antoine, Ann. Math. (2) 53 (1951) 276–297.

[5] K. Borsuk, An example of a simple arc in space whose projection in every plane has interior points, Fund. Math. 34 (1947) 272–277.

[6] J. Cobb, Raising dimension under *all* projections, Fund. Math. 144 (1994), 2, 119–128.

[7] O. Frolkina, A Cantor set in with "large" projections, Topol. Appl. 157 (2010), 4, 745–751.

[8] D.R. McMillan, Jr., Taming Cantor sets in E^n, Bull. Amer. Math. Soc. 70 (1964) 706–708.

[9] R.B. Sher, Concerning wild Cantor sets in E^3, Proc. Amer. Math. Soc. 19 (5) (1968) 1195–1200.

Motion Separation in a Neighborhood of a Semistable Cycle

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Let's consider an arbitrary C^∞ system F of ordinary differential equations in $\mathbb{R}^n$, $n \geq 2$, that has a periodic trajectory L_0 of the type of a simple saddle-node. Further, we introduce the two-dimensional system F_0 that is the restriction of the original system F to the center manifold $W^c(L_0)$ of the cycle L_0. We are interested in the simplest form to which the vector field F0 can be reduced in some sufficiently small neighborhood $\mathcal{U} \subset W^c(L_0)$ of the cycle L_0. We proof, that there exist local coordinates $(r, \psi) : |r| \leq r_0$, $r_0 = \text{const} > 0$, $0 \leq \psi \leq 2\pi \pmod{2\pi}$, in $\mathcal{U}$ in which the field F_0 acquires the form

$$F_0 = (r^2 + \alpha r^3)\partial/\partial r + \partial/\partial\psi,$$

where $\alpha \in \mathbb{R}$ is some constant. Therefore, we obtain a generalization of the well-known result due to Takens [1] on normal forms of scalar autonomous equations in a neighborhood of the zero equilibrium to the periodic case.

The so-called "blue sky catastrophe" is one possible application of the obtained results. By [2], the investigation of this bifurcation is essentially based on the existence of the change of variables reducing system F on the center manifold $W^c(L_0)$ to the normal form.

References

[1] Takens, F., Ann. Inst. Fourier, 1973, vol. 23, no. 2, pp. 163–195.

[2] Turaev, D.V. and Shil'nikov, L.P., Dokl. Akad. Nauk, 1995, vol. 342, no. 5, pp. 596–599.

The Ogasa Number

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Abstract: Given a manifold M, we look for the Morse functions with the "simplest" regular levels.

For this purpose, Ogasa introduces the following invariant. First of all, for any fixed Morse fonction f of M, one computes the sum of the Betti numbers of every regular level, then one keeps in mind only the maximum of these numbers. Hence, for all fixed f, this value depends on M and of f. Next, one minimises, letting f vary among all possible Morse functions of M. The value obtained from this minimax procedure depends only on the initial manifold M. It is the Ogasa invariant.

In dimension 2, the computation of this invariant is straightforward.

As for dimension 3, together with Michel Boileau (AMU, Marseille, France) we have understood what this dynamical invariant measures, and we have shown how it is related to other topological, geometric and algebraic invariants of the underlying manifold.

In higher dimension, the question is open.

In the talk, I will start with some examples of computation of this invariant, in order to become acquainted with it. Then I will focus on dimension 3: with the help of the obtained results, I shall explain why this dynamical invariant is so interesting.

Examples of Discrete Lorenz Attractors in Three-Dimensional Maps.

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In this talk we discuss dynamical properties and bifurcations of the so-called discrete Lorenz attractors [1,2]. Since these attractors appear as result of rather simple bifurcation scenarios [2,3], they can be often observed in various models (in particular, in those ones which are described by three-dimensional maps). We consider two such models: three-dimensional Hénon maps and nonholonomic models of Celtic stone.

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References

[1] S.Gonchenko, I.Ovsyannikov, C.Simo, D.Turaev, Three-dimensional Henon-like maps and wild Lorenz-like attractors // Int. J. of Bifurcation and chaos, 2005, v.15(11), 3493-3508.

[2] A.S. Gonchenko, S.V. Gonchenko, L.P. Shilnikov, Towards scenarios of chaos appearance in three-dimensional maps // Rus. Nonlinear Dyn., 2012, v.8(1), 3-28.

[3] A.S. Gonchenko, S.V. Gonchenko, Variety of strange pseudohyperbolic attractors in three-dimensional generalized Henon maps // Physica D, 2016, v. 337, 43-57.

On New Type of 1:4 Resonance in the Conservative Cubic Hénon Maps

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We study the 1:4 resonance for the conservative cubic Hénon maps $\mathbf{C}_{\pm}$ with positive and negative cubic term. These maps show up different bifurcation structures both for fixed points with eigenvalues $\pm i$ and for 4-periodic orbits. While for $\mathbf{C}_{-}$ the 1:4 resonance unfolding has the so-called Arnold degeneracy (the first Birkhoff twist coefficient equals (in absolute value) to the first resonant term coefficient), the map $\mathbf{C}_{+}$ has a different type of degeneracy because the resonant term can vanish. This new type of degeneracy has not been studied before. In this case, non-symmetric points are created and destroyed at pitchfork bifurcations and, as a result of global bifurcations, the 1:4 resonant chain of islands rotates by $\pi/4$. For both maps several bifurcations are detected and illustrated. This is a joint work with S. Gonchenko, I. Ovsyannikov and A. Vieiro.

On Bifurcations of a 3-parameter Family of Two-dimensional Diffeomorphisms with a Quadratic Homoclinic Tangency to a Nonhyperbolic Saddle at $\mu = 0$

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We consider a C^r-diffeomorphism f_0 $(r \geq 4)$, satisfying the following conditions:

A) f_0 has a fixed point O, which is a nonhyperbolic saddle with the multipliers $\lambda_1 = \lambda$, where $0 < |\lambda| < 1$, $\lambda_2 = 1$, and with the second Lyapunov value $l_3 > 0$.

B) Invariant manifolds $W^u(O)$ and $W^s(O)$ of O have a (single-round) quadratic tangency at the points of a homoclinic orbit Γ_0.

The conditions A)–B) define a codimension 3 locally connected bifurcation surface in the space of two-dimensional diffeomorphisms, and, hence, for studying bifurcations of f_0, we must consider 3-parameter families. Let f_μ, where $\mu = (\mu_1, \mu_2, \mu_3)$, be such a family, which unfolds degenarations, given by the conditions A)–B).

Theorem 1. Theorem 1 *Let U be a sufficiently small neighborhood of the origin in the (μ_1, μ_2, μ_3)-parameter space. Then in U there is a two-dimensional discontinuous surface S (with the edge on the curve Γ_1^*: $\mu_3 = (\frac{\mu_1}{2})^{\frac{1}{3}}, \mu_2 = -3(\frac{\mu_1}{2})^{\frac{2}{3}}$), such that f_μ has a (single-round) quadratic homoclinic tangency either to a hyperbolic saddle fixed point or to a saddle-node fixed point, when $\mu \in \Gamma_1^*$.*

Theorem 2. Theorem 2 *In the (μ_1, μ_2, μ_3)-parameter space, in any sufficiently small neighborhood $U(\mu = 0)$ there is infinitely many nonintersecting domains Δ_k, such that at $\mu \in \Delta_k$ the diffeomorphism has an asymptotically stable single-round periodic orbit.*

1) Boundaries of Δ_k correspond to codimension 1 bifurcations for single-round periodic orbits — saddle-node and period doubling ones.

2) Domains Δ_k accumulate to the surface S as $k \to +\infty$.

Embedding in Flows of Morse-Smale Cascades

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J. Palis found necessary conditions for a Morse-Smale diffeomorphism on a closed n-dimensional manifold M^n to embed into a topological flow and proved that these conditions are also sufficient for $n = 2$. For the case $n = 3$ a possibility of wild embedding of closures of separatrices of saddles is an additional obstacle for Morse-Smale cascades to embed into topological flows. We show that there are no such obstructions for Morse-Smale diffeomorphisms without heteroclinic intersection given on the sphere S^n, $n \geq 4$, and Palis's conditions again are sufficient for such diffeomorphisms.

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Boundary Deformation Rate in Discrete and Continuous Time Dynamical Systems

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A relation between the boundary deformation rate in the phase space of a dynamical system and the metric entropy of the system was first observed in physics literature [3], [4]. Mathematically, this can be stated as follows. Let $\{S^t\}$ be a dynamical system with discrete or continuous time acting in a metric space (X, ρ) and μ an $\{S^t\}$-invariant Borel probability measure. Denote by $B = B(x, \varepsilon)$ the ball of radius ε centered at $x \in X$ and define the local deformation rate by

$$R_t(B, x, \varepsilon) := \frac{\mu(\mathcal{O}_\varepsilon(S^t B))}{\mu(B)}, \tag{1}$$

where $O_\varepsilon(S^t B)$ is the ε-neighborhood of $S^t B$. Let $h_\mu(\{S^t\})$ denote the entropy of $\{S^t\}$ with respect to μ. We formulate conditions under which

$$\lim_{\varepsilon \to 0} \frac{1}{t} \log R_t(B, x, \varepsilon) = h_\mu(\{S^t\}) \tag{2}$$

(the convergence can be taken in various senses). A general condition is that t depends on ε in such a way that $t(\varepsilon) \to \infty$ and $t(\varepsilon)/\log \varepsilon \to 0$ as $\varepsilon \to 0$. If this condition holds, then (2) is true for some symbolic dynamical systems among which are subshifts of finite type, sofic systems, and synchronized systems [1], [2]. Moreover, one can assume that t in (1) depends not only on ε, but on x, the center of the ball, as well. We impose the following restriction on this dependence: there exists a function $\tau : (0,1) \to \mathbb{N}$ such that

$$\lim_{\varepsilon \to 0} \tau(\varepsilon) = \infty, \quad \lim_{\varepsilon \to 0} \tau(\varepsilon)/\log \varepsilon = 0,$$

$$\lim_{\varepsilon \to 0} \frac{t(\varepsilon, x)}{\tau(\varepsilon)} = 1$$

for μ-almost all x, and

$$a\tau(\varepsilon) < t(\varepsilon, x) < b\tau(\varepsilon)$$

for all ε and μ-almost all x, where $a, b > 0$ do not depend on ε and x. In this situation (2) is also true (with L^1_μ-convergence).

This result makes it possible to consider a class of continuous time systems. These are suspension flows over the symbolic systems (maps in the base) mentioned above. If the roof function of the flow satisfies some regularity condition, then (2) holds except that the measure μ should be repalaced by the flow invariant measure determined by μ.

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References

[1] B.M. Gurevich, Geometric interpretation of entropy for random processes // *Sinai's Moscow Seminar on Dynamical Systems*, Providence, R.I.: Amer. Math. Soc. 1996, 81–87.

[2] S.A. Komech, Boundary distortion rate in synchronized systems: geometrical meaning of entropy (Russian), *Problemy Peredachi Informatsii* **48**:1 (2012), 15-25; English translation: *Probl. Inf. Transm.* **48**:1 (2012), 11-20.

[3] G.M. Zaslavsky, *Stochasticity of dynamical systems* (Russian), Moscow, Nauka, 1984.

[4] G.M. Zaslavsky *Hamiltonian Chaos and Fractal Dynamics*, Oxford, Oxford University Press, 2005.

Liouville Type Theorems for Transversally Harmonic Map

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Transversally harmonic map (resp. biharmonc map) from the foliated manifold $(M,\mathcal{F})$ to $(M',\mathcal{F}')$ is a smooth leaf-preserving map such that the transvresal tension (resp. *bitension*) field vanishes. The classical Liouville theorem says that any bounded harmonic function defined on the whole plane must be constant. The classical Liouville theorem has been improved in several cases. In this talk, we give the Liouville type theorems for transversally harmonic and biharmonic maps on foliated Riemannian manifolds. That is, under the assumption of non-negative transversal Ricci curvature of $\mathcal{F}$ and nonpositive transversal sectional curvature of $\mathcal{F}'$, any transversally harmonic map ϕ: $(M,\mathcal{F}) \to (M',\mathcal{F}')$ of finite transversal energy is transversally constant, i.e., the induced map between leaf spaces is constant. And any transversally biharmonic map of finite transversal bienergy is transversally harmonic.

Multibump Trajectories of Periodic Lagrangian Systems with Pitchfork Bifurcations

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We study a Hamiltonian system with the Hamiltonian

$$H(q,p,\varepsilon t) = \frac{p^2}{2} - \frac{1}{2}\varphi(\varepsilon t)q^2 + \frac{1}{4}q^4, \quad \varepsilon \ll 1, \tag{1}$$

slowly depending on time. It is assumed the function φ to be a periodic C^2-function with period 1, which satisfy the following condition
(H_1): there exist $\tau_k \in [0,1), k = 1,\dots,2m$ such that $\varphi(\tau_k) = 0$, $\varphi'(\tau_k) \neq 0$.

Such a system can be considered as the simplest example of the slow-fast systems. The Hamilton's equations associated to (1) and considered in the extended phase space are

$$\dot{q} = p, \quad \dot{p} = \varphi(\tau)q - q^3, \quad \dot{\tau} = \varepsilon.$$

Thus, (q,p) can be treated as fast variables and τ as a slow one. The "slow" manifold Γ of this system is defined by equations $p = 0, \varphi(\tau)q - q^3 = 0$ and consists of one point $(q,p) = (0,0)$ if

$\varphi(\tau) \le 0$ and of three points $(0,0), (\pm\sqrt{\varphi(\tau)})$ in the case $\varphi(\tau) > 0$. Such pitchfork bifurcation occurs each time the parameter τ passes through the point τ_k. When $\varphi(\tau) > 0$ the origin becomes a hyperbolic equilibrium of the " frozen" system and possesses the figure-eight separatrix with two branches.

The system (1) describes the motion of a charged particle in the Earth's magnetospheric tail and was extensively studied in many papers. In the present work we show that the sequence of bifurcations leads to appearence of multi-bump trajectories of the system. In particular we show that there exists $\varepsilon_0 > 0$ and a subset $\mathcal{E}_h \subset (0, \varepsilon_0)$ such that
1. for any $\varepsilon_1 < \varepsilon_0$ the Lebesgue measure $leb\left((0,\varepsilon_1) \setminus \mathcal{E}_h\right) = O\left(\mathrm{e}^{-c/\varepsilon_1}\right)$ with some positive constant c;
2. for any $\varepsilon \in \mathcal{E}_h$ the origin is a hyperbolic equilibrium of the system (1);
3. for any sequence $\{a_k\}_{k=1}^N$ with $a_k \in \{-1, 0, 1\}$ there exists a multi-bump trajectory which stays during the time between τ_k and τ_{k+2} in a small neighborhood of the origin if $a_k = 0$ or follows one of the branches of the "frozen" separatrix ($a_k = \pm 1$).

Effective Algebraic Geometry and Control Theory: Applications to Medical Problems

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The goal of Effective Algebraic Geometry is to solve algebraic and semi-algebraic problems using constructive proofs and methods.

This way of solving problems is worth of interest in various questions arising in Optimal Control Theory.

We shall consider here different issues coming from Nuclear Magnetic Resonance Imagery as well as other challenging medical problems.

In the particular case of NMRI, the understanding of the underlying geometry of the optimality conditions for the control problem is obtained by solving exactly algebraic and semi-algebraic systems. These studies have then been completed by numerical computations for different test cases, and finally confirmed by in vitro and in vivo experiments.

Fejer Sums and the von Neumann Ergodic Theorem

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The Fejer sums for periodical measures and the norms of the deviations from the limit in the von Neumann ergodic theorem both are calculating, in fact, with the same formulas (by integrating of the Fejer kernels) — and so, this ergodic theorem, in fact, is a statement about the asymptotic of the growth of the Fejer sums at zero point of the spectral measure of corresponding dynamical system. It gives a possibility to rework well-known estimates of the rates of convergence in the von Neumann ergodic theorem into the estimates of the Fejer sums in the point for periodical measures — for example, we obtain natural criteria of polynomial growth and polynomial degree of these sums.

And vice versa, numerous in the literature estimates of the deviations of Fejer sums in the point allow to obtain estimates of the rate of convergence in this ergodic theorem. For example, we obtain from the results of S.N. Bernstein in harmonic analysis more than hundred years old the estimates of the rates of convergence in the von Neumann ergodic theorem for many popular in the applications dynamical systems — with sharp senior coefficient of the asymptotic.

Convergence of Spherical Averages for Actions of Fuchsian Groups

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The talk is based on the joint work [1] with A. Bufetov and C. Series.

Consider an action of a Fuchsian group G on a Lebesgue probability space (X, μ) by measure-preserving maps T_g. Let $\mathcal{R}$ be a fundamental domain of G and let G_0 be the symmetric generating set of G that consists of all elements h such that $\mathcal{R}$ and $h\mathcal{R}$ share a common side. Assume that $\mathcal{R}$ has *even angles*, that is, the boundary of the tessellation $\{g\mathcal{R}\}$ is a union of complete geodesic lines. Finally, for $\varphi \in L^1(X, \mu)$ let us define its spherical averages as follows:

$$\mathbf{S}_n(\varphi) = \frac{1}{\#\{g : |g| = n\}} \sum_{g:|g|=n} \varphi \circ T_g,$$

where $|g|$ is a norm of $g \in G$ with respect to the generating set G_0.

Theorem. *Let G be a non-elementary Fuchsian group G and suppose it has a fundamental domain $\mathcal{R}$ with even corners and satisfying a technical condition (in particular, it holds for any domain with at least 5 sides).*

Let G act on a Lebesgue probability space (X, μ) by measure-preserving transformations T_g. Denote by $\mathcal{I}_{G_0^2}$ the σ-algebra of sets invariant under all maps $T_{g_1 g_2}$, $g_1, g_2 \in G_0$, where G_0 is defined above. Then for any function $f \in L \log L(X, \mu)$, the sequence $(\mathbf{S}_{2n}(f))$ converges as $n \to \infty$ almost surely and in L^1 to the conditional expectation $\mathbb{E}(f | \mathcal{I}_{G_0^2})$ with respect to the σ-algebra $\mathcal{I}_{G_0^2}$.

The proof is based on the construction of the Markov coding for the group G with the following symmetry property. We construct a topological Markov chain with a finite set of states Ξ, two maps $\gamma, \omega \colon \Xi \to G_0$, and subsets $\Xi_F, \Xi_S \subset \Xi$ such that the map

$$(i_0 \to i_1 \to \cdots \to i_{n-1}) \mapsto \omega(i_{n-1})\gamma(i_{n-2}) \ldots \gamma(i_0)$$

is a bijection from the set of all admissible sequences of states with $i_0 \in \Xi_F$, $i_{n-1} \in \Xi_S$ to the sphere $\{|g| = n\}$ in the group G. The symmetry property states that there exists an involution $\iota \colon \Xi \to \Xi$ such that

- it reverses the time of our Markov chain, i. e. transitions $j \to k$ and $\iota(k) \to \iota(j)$ are admissible simultaneously;
- the identities $\omega(\iota(k)) = \omega(k)^{-1}$ and $\omega(j)\gamma(k) = \gamma(\iota(j))^{-1}\omega(k)$ hold for all k and all admissible transitions $k \to j$ respectively;
- and therefore, if an element $g \in G$ corresponds to the sequence $(i_0 \to \cdots \to i_{n-1})$, the inverse element g^{-1} corresponds to the sequence $(\iota(i_{n-1}) \to \cdots \to \iota(i_0))$.

References

[1] A. I. Bufetov, A. Klimenko and C. Series, Convergence of spherical averages for actions of Fuchsian Groups, preprint, `arXiv:1805.11743`.

Hochschild Cohomology via Morse Mathching and Anick Resolution

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Homological methods allow us to get important information about the structure of an algebra. For associative algebras, Hochschild cohomologies play an important role in structure and representation theory. Finding the Hochschild cohomology group $H^n(A, M)$ of a given algebra A with coefficients in a given A-bimodule M is often a difficult problem. In order to solve this problem one needs a long exact sequence starting from A, a resolution of A. The most natural bar-resolution is easy to construct but it is too bulky for computations. Another approach was proposed by David J. Anick in 1986 [1], where it was built a free resolution for associative algebra which is homotopy equivalent to the bar-resolution. The Anick resolution was also used to find Poincare Series. Computation of the differentials in the Anick resolution according to the original algorithm described in [1] is extremely hard. In order to make the computation easier, one may use the discrete algebraic Morse theory based on the concept of a Morse matching defined in [3, 4]. This concept was used in geometry first, then it became applicable in algebra. In the present work, we apply the Morse matching theory to find the Anick resolution and calculate the groups of Hochschild n-cohomologies of the Manturov group (which has applications in Dynamical Systems[6]), Weyl algebra and chinese algebra.

References

[1] D. J. Anick. On the homology of associative algebras, Transactions of the American Mathematical Society, 1986, Vol. 296, No 2, P. 641-659.

[2] V. Lopatkin. Cohomology rings of the plastic monoid algebra via a Grobner - Shirshov basis, 2017, arXiv:1411.5464v7 [math.AT].

[3] E. Sköldberg, Morse theory from an algebraic viewpoint, *Trans. Amer. Math. Soc.*, 358 (1) (2006) 115 – 129.

[4] M. Jöllenbeck and V. Welker, *Minimal Resolutions Via Algebraic Discrete Morse Theory*, Mem. Am. Math. Soc. 197 (2009).

[5] Yuxiu Bai and Yuqun Chen, Gröbner-Shirshov Bases for the Groups G_3^2 and G_4^3, School of Mathematical Sciences, South China Normal University, (2017).

[6] V.O.Manturov, Non-reidemeister knot theory and its applications in dynamical sestems, geometry, and topology, preprint (2015), arXiv:1501.05208.

Laplacians on Generalized Smooth Distributions

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We will discuss the Laplacians associated with an arbitrary generalized smooth distribution on a smooth manifold M. Roughly, smooth distributions are smooth assignments of vector subspaces D_x of T_xM for every $x \in M$. These subspaces are not required to have constant rank. It is more convenient to view such a distribution in terms of its dynamics, focusing on the module $\mathcal{D}$ of smooth vector fields on M tangent to it. More precisely, we define a (generalized) smooth distribution on M as a locally finitely generated $C^\infty(M)$-submodule $\mathcal{D}$ of the $C^\infty(M)$-module $\mathcal{X}_c(M)$ of smooth compactly supported vector fields on M. If the distribution has constant rank, D is a subbundle of the tangent bundle TM and the associated $C^\infty(M)$-module $\mathcal{D}$ is the space of smooth sections of this bundle: $\mathcal{D} = C^\infty(M, D)$. In this case $\mathcal{D}$ is projective.

The fiber of the distribution $\mathcal{D}$ at $x \in M$ is the finite dimensional vector space $\mathcal{D}_x = \mathcal{D}/I_x\mathcal{D}$, where $I_x = \{f \in C^\infty(M) : f(x) = 0\}$. We define a Riemannian metric on $\mathcal{D}$ as a family of inner products $\langle\ ,\ \rangle_x$ on $\mathcal{D}_x$, depending smoothly on $x \in M$ in some sense. We prove that such a Riemannian metric exists for an arbitrary distribution $\mathcal{D}$.

Given a smooth distribution $\mathcal{D}$ on a smooth manifold M, a Riemannian metric on $\mathcal{D}$ and a positive density μ on M, we construct the associated horizontal Laplacian as follows. First, we define the horizontal differential to be the operator $d_{\mathcal{D}} : C_c^\infty(M) \to C_c^\infty(M, \mathcal{D}^*)$ given by $d_{\mathcal{D}} = ev^* \circ d$, where $d : C_c^\infty(M) \to \Omega_c^1(M)$ is the de Rham differential and $ev^* : \Omega_c^1(M) \to C_c^\infty(M, \mathcal{D}^*)$ is induced by the evaluation maps $ev_x : \mathcal{D}_x \to T_xM$, $x \in M$. The horizontal Laplacian of $\mathcal{D}$ is the second order differential operator $\Delta_{\mathcal{D}} = d_{\mathcal{D}}^* \circ d_{\mathcal{D}} : C_c^\infty(M) \to C_c^\infty(M)$, where $d_{\mathcal{D}}^* : C_c^\infty(M, \mathcal{D}^*) \to C_c^\infty(M)$ is the adjoint of $d_{\mathcal{D}}$ with respect to natural inner products on $C_c^\infty(M)$ and $C_c^\infty(M, \mathcal{D}^*)$ defined by the Riemannian metric on $\mathcal{D}$ and the density μ. We show that, if M is compact, the horizontal Laplacian $\Delta_{\mathcal{D}}$ as an unbounded operator on the Hilbert space $L^2(M, \mu)$ with domain $C^\infty(M)$ is essentially self-adjoint.

A distribution $\mathcal{D}$ is called involutive, if it is closed under Lie brackets: $[\mathcal{D}, \mathcal{D}] \subseteq \mathcal{D}$. An involutive smooth distribution is called a singular foliation. I. Androulidakis and G. Skandalis constructed a longitudinal pseudodifferential calculus and the corresponding scale of longitudinal Sobolev spaces for an arbitrary singular foliation on a compact manifold.

For a smooth distribution $\mathcal{D}$ on a compact manifold M, consider the smallest involutive $C^\infty(M)$-submodule $\mathcal{F}$ of $\mathcal{X}(M)$, which contains $\mathcal{D}$. It is generated by the elements of $\mathcal{D}$ and their iterated Lie brackets $[X_1, \dots, [X_{k-1}, X_k]]$ such that $X_i \in \mathcal{D}$, $i = 1, \dots, k$, for every $k \in N$. Assume that $\mathcal{F}$ is a singular foliation (that is, it is finitely generated). We prove that the horizontal Laplacian $\Delta_{\mathcal{D}}$ is longitudinally hypoelliptic in the scale of longitudinal Sobolev spaces associated with $\mathcal{F}$.

This is joint work with I. Androulidakis.

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On Asymptotic Behavior of Solutions for Inclusions with Causal Multioperators

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The study of systems governed by differential and functional equations with causal operators, which is due to Tonelli [1] and Tychonov [2], attracts the attention of many researchers. The term causal arises from the engineering and the notion of a causal operator turns out to be a powerful tool for unifying problems in ordinary differential equations, integro-differential equations, functional differential equations with finite or infinite delay, Volterra integral equations, neutral functional equations et al. (see the monograph [3]).

The main ideas of the method of guiding functions were formulated by Krasnosel'skii and Perov in the fifties (see [4]). In the recent years the method of integral guiding functions became one of the most significant directions in the developments of the guiding functions theory (see, e.g., [5]).

In the present talk we apply the method of non-smooth integral guiding potentials to the investigation of the asymptotic behavior of solutions for a differential inclusion with the multivalued causal operator.

Other aspects of the method of guiding functions and its applications, as well as the additional bibliography, may be found in the recent monograph [6].

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References

[1] Tonelli, L. Sulle equazioni funzionali di Volterra. Bull. Calcutta Math. Soc. 20 (1930), 31–48.

[2] Tikhonov, A.N. Functional Volterra-type equations and their applications to certain problems of mathematical physics (in Russian), Bull. Mosk. Gos. Univ. Sekt. A, 1(1938), no. 8, 1–25.

[3] Corduneanu, C. Functional Equations with Causal Operators. Stability and Control: Theory, Methods and Applications, 16. Taylor and Francis, London, 2002.

[4] Krasnosel'skii, M.A. The Operator of Translation Along the Trajectories of Differential Equations. Translations of Mathematical Monographs, Vol. 19. American Mathematical Society, Providence, R.I. 1968.

[5] Kornev, S.; Obukhovskii, V.; Zecca, P. Guiding functions and periodic solutions for inclusions with causal multioperators. Appl. Anal. 96 (2017), Issue 3, 418–428.

[6] Obukhovskii, V.; Zecca, P.; Loi, N.V.; Kornev, S. Method of Guiding Functions in Problems of Nonlinear Analysis. Lecture Notes in Mathematics, 2076. Springer, Heidelberg, 2013.

Chaotic Regimes in the Ensemble of FitzhHugh-Nagumo Elements with Weak Couplings

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We study the peculiarities of chaotic dynamics in the phenomenological model of ensemble of two FitzHugh-Nagumo elements with weak excitatory couplings. This model was recently proposed as a suitable model for describing the behaviour of two coupled neurons. A rich diversity of different types of neuron-like behaviour, including regular in-phase, anti-phase, sequential spiking activities and also chaotic activity was observed in this model. We focus on chaotic bursting and chaotic spiking neuron-like activity in this paper. We study in details bifurcation scenarios of the emergence and destruction of these types of neuron-like activity

On the Dynamics of the Pendulum Equation with Asymmetric Quasi-periodic Perturbations

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Quasi-periodic two-frequency perturbations of an asymmetric pendulum type equation close to an integrable one are considered. The structures of the resonance zones are studied. The conditions for the existence of resonance quasi-periodic solutions (two-dimensional resonance tori) are found.

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On the Dynamics of Rough 3-diffeomorhisms with 2-dimensional Expanding Attractor

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The results of this paper were obtained in collaboration with V.Z Grines and O.V. Pochinka.

Let $f : M^3 \to M^3$ be a structurally stable diffeomorphism such that its nonwandering set contains an expanding orientable attractor Λ of topological dimension 2. We denote the class of such diffeomorphisms by G. A point $x \in \Lambda$ is called boundary if one of the connected components of the set $W^s(x)\backslash x$ does not intersect with Λ, denote this component by $W^{s\emptyset}(x)$. The set Γ_Λ of all boundary points of the set Λ is nonempty and consists of finite number of periodic points that are divided into associated couples (p, q) of points of the same period so the 2-bunch $B_{pq} = W^u(p) \cup W^u(q)$ is a boundary achievable from inside of the connect component of the set $M\backslash\Lambda$. Then the following facts take place: (1) the ambient manifold M^3 is homeomorphic to the 3-dimensional torus T^3; (2) for each associated couple (p, q) of boundary points there exists a natural number k_{pq} such that the set of nonwandering points of diffeomorphism f contains k_{pq} periodic sources $\alpha_1,...,\alpha_{k_{pq}}$ and $k_{pq}-1$ periodic saddle points $P_1,...,P_{k_{pq}-1}$ alternate on the simple arc $l_{pq}=W^{s\emptyset}(p)\cup\bigcup\limits_{i=1}^{k_{pq}-1}W^s(P_i)\cup\bigcup\limits_{i=1}^{k_{pq}}W^s(\alpha_i)\cup W^{s\emptyset}(q)$ [1]. Denote the separatrices $\ell_{i-1}^{\alpha_i}=W^s(P_{i-1})\cap W^u(\alpha_i)$, $\ell_i^{\alpha_i}=W^s(P_i)\cap W^u(\alpha_i)$, $i=1,2,...,k_{pq}$; $P_0=p$, $P_{k_{pq}}=q$ and m_{α_i} is period of the separatrices $\ell_{i-1}^{\alpha_i}$, $\ell_i^{\alpha_i}$. We are called the union $F^{\alpha_i}=\ell_{i-1}^{\alpha_i}\cup\alpha_i\cup\ell_i^{\alpha_i}$, $i=1,2,...,k_{pq}$, is a associated with the source α_i frame of two separatrices.

Theorem 1. *Let $f\in G$, (p,q) is associated pair of boundary points p, q. Then separatrises $\ell_{i-1}^{\alpha_i}$, $\ell_i^{\alpha_i}$ and frames F^{α_i}, $i=1,2,...,k_{pq}$, are tamely embedded.*

As α_i is a hyperbolic point then the diffeomorphism $f^{m_{\alpha_i}}|_{W^u(\alpha_i)}$ is topologically conjugated with the homothety $A:R^3\to R^3$ defined by $A(x_1,x_2,x_3)=(2x_1,2x_2,2x_3)$ by means a homeomorphism $h_{\alpha_i}:W^u(\alpha_i)\to R^3$. Let $J_2^{\alpha_i}=\pi(F_2^{\alpha_i})$ where $\pi:W^u(\alpha_i)\to S^2\times S^1$. Then $J_2^{\alpha_i}$ is a frame of two circles in $S^2\times S^1$, which we call associated with the source α_i.

Theorem 2. *For every periodic source α_i of a diffeomorphism $f\in G$ the associated with α_i frame of circles $J_2^{\alpha_i}$ is trivial.*

References

[1] Grines V., Zhuzhoma E. On structurally stable diffeomorphisms with codimension one expanding attractors// Trans. Am. Math. Soc. – 2005.– 357, 2.– P. 617-667.

On Moduli for Gradient Surface Height Function Flows

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Two flows are called topologically equivalent if there is a homeomorphism sending trajectories of one flow into trajectories of another one preserving directions of trajectories. Two flows are called topologically conjugate if they are topologically equivalent by means of the homeomorphism preserving time of moving along trajectories.

In 1978 J. Palis [1] invented continuum topologically non-conjugate systems in a neighbourhood of a system with a heteroclinic contact (moduli). W. de Melo and C. van Strien in 1987 [2] described a diffeomorphism class with a finite number of moduli: a chain of saddles taking part in the heteroclinic contact of such diffeomorphism includes not more than three saddles. Surprisingly, such effect does not happen in flows. Here we consider gradient flows of the height function for an orientable surface of genus $g>0$. Such flows have a chain of $2g$ saddles.

In this research we speak about the class G of C^2-smooth gradient flows $f^t:S_g\to S_g$ induced by gradient vector field of the height function for vertical orientable surface S_g of genus $g>0$. The non-wandering set of such systems consists of a finite value of hyperbolic fixed points: a single source, a single sink and a finite number of saddle points constructing a chain with each element connected with the next one by two saddle separatrices.

The main result of our work is the next theorem

Theorem. *Any flow $f^t : S_g \to S_g$ from the class G has exactly $2g-1$ moduli.*

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References

[1] J. Palis, "A Differentiable Invariant of Topological Conjugacies and Moduli of Stability", *Astérisque* **51** (1978), 335-346

[2] W. de Melo and S. J. van Strien, "Diffeomorphisms on surfaces with a finite number of moduli", *Ergodic Theory and Dynamical Systems* **7** (1987), 415-462

First Steps of the Global Bifurcation Theory in the Plane

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The talk is devoted to a part of a new theory: global bifurcations in the plane. The central question is: who bifurcates. The answer, obtained together with N. Goncharuk, may plan the key role in many bifurcation problems in the future.

Implementation of Smale–Williams Solenoids with Different Expansion Factors

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We propose a new class of systems demonstrating Smale–Williams solenoids. The principle of their constructing is based on using self-oscillatory subsystems with resonant transfer of oscillatory excitation between them. Smale–Williams solenoids arize due to the difference in the frequencies of small and large (relaxation) oscillations by an integer number of times being accompanied by phase transformation according to an expanding circle map. We introduce three physical systems of proposed class.

At first we discuss a model of a new class that consists of two weakly coupled Bonhoeffer–van der Pol oscillators with periodically modulated parameters. Oscillators alternatively undergo smooth transition from small self-oscillations to relaxation self-oscillations. Depending on parameters, the Smale–Williams solenoids with different expansion factors M of the angular variable appear in phase space of the system. If one of the parameters is zero, the model becomes a system of coupled van der Pol oscillators, in which Smale–Williams solenoids also arise. The equations are:

$$\begin{aligned} \dot{x} = u, \quad \dot{u} = \left(f\left(\tfrac{t}{T} + \tfrac{1}{4}\right) - x^2\right) u - x + K + \varepsilon\,(y - x)\,, \\ \dot{y} = v, \quad \dot{v} = \left(f\left(\tfrac{t}{T} - \tfrac{1}{4}\right) - y^2\right) v - y + K + \varepsilon\,(x - y)\,, \end{aligned} \tag{1}$$

where T is modulation period, ε is coupling parameter, K is constant parameter. Function $f(t)$ describes modulation:

$$f(\tau) = f(\tau + 1) = \begin{cases} a, & \text{if } 0 \leq \tau < \tau_1, \\ \frac{(a-c)\tau + c\tau_1 - a\tau_2}{\tau_1 - \tau_2}, & \text{if } \tau_1 \leq \tau < \tau_2, \\ \frac{(c-a)\tau + a\tau_2 - c}{\tau_2 - 1}, & \text{if } \tau_2 \leq \tau < 1. \end{cases} \tag{2}$$

We also introduce another model which is physically similar to the previous system. It is composed of two weakly coupled FitzHugh–Nagumo neurons with periodical modulation of parameters:

$$\begin{array}{ll} \dot{x} = f\left(\frac{t}{T} + \frac{1}{4}\right) x - \frac{1}{3}x^3 - u + \varepsilon(y - x), & \dot{u} = ax - bu + I, \\ \dot{y} = f\left(\frac{t}{T} - \frac{1}{4}\right) y - \frac{1}{3}y^3 - v + \varepsilon(x - y), & \dot{v} = ay - bv + I, \end{array} \tag{3}$$

where T is modulation period, ε is coupling parameter, I, a and b are constant parameters. Function $f(t)$ is described by (2).

The third example is composed of two coupled Froude pendulums placed on a common shaft rotating at constant angular velocity with braking by application of frictional force to one and other pendulum turn by turn periodically:

$$\begin{array}{ll} \dot{x} = u, & \dot{u} = \left(a - d(t) - bu^2\right) u - \sin x + \mu + \varepsilon(v - u), \\ \dot{y} = v, & \dot{v} = \left(a - d\left(t + \frac{T}{2}\right) - bv^2\right) v - \sin y + \mu + \varepsilon(u - v), \end{array} \tag{4}$$

where T is modulation period, ε is coupling parameter, μ, a and b are constant parameters. Function $d(t)$ describes periodic dumping:

$$d(t) = \begin{cases} 0, \ t < T_0, \\ D, \ T_0 < t < T/2, \\ 0, \ T/2 < t < T. \end{cases} \qquad d(t + T) = d(t). \tag{5}$$

Proposed systems were studied numerically. It was demonstrated that Smale–Williams solenoids with different expanding factors appear in Poincaré stroboscopic maps on specific intervals of parameter values. The hyperbolicity of chaotic attractors was confirmed with the help of a test based on numerical evaluation of angles of intersections of stable and unstable manifolds of attractor with verification of the absence of tangencies between these manifolds.

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References

[1] Doroshenko V. M., Kruglov V. P., Kuznetsov S. P. Smale–Williams Solenoids in a System of Coupled Bonhoeffer–van der Pol Oscillators //Russian Journal of Nonlinear Dynamics, vol. 14, no. 4, pp. 435-451, 2018.

[2] Kuznetsov S. P., Kruglov V. P. Hyperbolic chaos in a system of two Froude pendulums with alternating periodic braking //Communications in Nonlinear Science and Numerical Simulation. vol. 67. pp. 152-161, 2019.

Controllability and Infinite Invariant Measures

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In the recent paper by D. Burago, S. Ivanov and A. Novikov, — A survival guide for feeble fish —, it has been shown that a fish with limited velocity can reach any point in the (possibly unbounded) ocean provided that the fluid velocity field is incompressible, bounded and has vanishing mean drift. This result extends some known global controllability theorems though being substantially nonconstructive. We give a fish a different recipe of how to survive in a turbulent ocean, and show its relationship to structural stability of dynamical systems by providing a constructive way to change slightly the velocity field to produce conservative (in the sense of not having wandering sets of positive measure) dynamics. In particular, this leads to the extension of Ch. Pugh's closing lemma to incompressible vector fields over unbounded domains. The results are based on an extension of the Poincaré recurrence theorem to some σ-finite measures and on specially constructed Newtonian potentials.

Also, we provide a discrete version of our results and prove that systems with small mean drifts satisfy many properties of ones with a probability invariant measure.

On Existence of Expanding Attractors of Endomorphisms of 2-torus

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In 1967 S. Smale proposed a method how to construct hyperbolic codimension one attractors and repellers of diffeomorphisms given on n-torus. The construction was based on Anosov toral algebraic diffeomorphism. In essence, he described local modification of such automorphism that leads to so called derived from Anosov diffeomorphism. The nonwandering set of DA-diffeomorphism consists either of an expanding codimension one attractor and a trivial source or of codimension one contracting repeller and a trivial sink. In the present talk we show that Smale's surgery operation applied to Anosov endomorphism that is k-fold covering map of degree not less than two does not lead to an A-endomorphism with one-dimensional expanding attractor.

Theorem. *Let $f\colon \mathbb{T}^2 \to \mathbb{T}^2$ be A-endomorphism, which is k-fold covering, $k \geq 2$. Then f can not have one-dimensional attractor Λ, such that:*

- Λ *is strictly invariant;*
- *unstable manifold $W^u(x)$ of each point $x \in \Lambda$ does not depend on the trajectory through point x and forms a one-dimensional curve, which is not closed;*
- *the following equality holds $\bigcup_{x\in\Lambda} W^u(x) = \Lambda$ and, Λ forms a lamination locally homeomorphic to the product of a Cantor set with an interval;*
- *attainable boundary of any connected component of the set $\mathbb{T}^2 \setminus \Lambda$ consists of the finite number of leafs of Λ.*

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Nonalternating Hamiltonian Lie (super)Algebras in Characteristic Two and Related Topics (Deformations of Lie algebras, Non-degenerate Derivations and p-Groups)

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Simple finite-dimensional Lie algebras of characteristic $p > 0$ naturally arise in the theory of p-groups and pro-p-groups, as the Lie algebras associated with the filtrations $\{G_i\}$ such that $(G_i, G_j) \subset G_{i+j}$. For important classes of groups, under suitable filtration, Lie algebras form a positive part of the twisted affine Lie algebra, $L = L(\mathfrak{g}, \alpha, m) = \oplus_{i>0}\mathfrak{g}_{\bar{i}} \otimes t^i$, where $\mathfrak{g} = \oplus_{\bar{i}\in\mathbb{Z}_m}\mathfrak{g}_{\bar{i}}$ is a grading mod m of a simple Lie algebra over a field of characteristic p. For the theory of p-groups of finite coclass, simple Lie algebras over a field of characteristic p that admit non-singular derivation are of interest [1], [2]. When $p > 3$, all such simple Lie algebras were found in [3]. These are special and Hamiltonian Lie algebras of vector fields corresponding to differential forms with a nonzero cohomology class. Note that the algebra of functions must be replaced by an algebra of divided powers $\mathcal{O}_n(\mathcal{F})$(see [4]) that corresponds to some generalized flag $\mathcal{F}$. In the case of a field of characteristic 2 one can construct a large class of simple generalized Hamiltonian Lie algebras corresponding to symmetric differential forms. The number of variables can also be odd. The class of Hamiltonian Lie algebras of characteristic 2 with the simplest symmetric Poisson bracket was constructed in 1993 by Lei Lin [5]. The authors of [6] - [8] noted that this bracket is obtained from the classical Poisson bracket of the Hamiltonian Lie superalgebra and proposed the construction of symmetric differential forms. In [9] the invariant construction of the complex of symmetric differential forms in characteristic 2 was given and some program of investigation was proposed. The authors have obtained all invariants of symmetric Hamiltonian differential forms with constant coefficients with respect to parabolic subgroup of $GL(V)$ corresponding to flag $\mathcal{F}$. In particular, it was shown that there exists a basis of V coordinated with flag $\mathcal{F}$ such that a symmetric Hamiltonian form has a matrix $diag(M_0, \dots, M_0,\ M_1, \dots, M_1,\ 1_s)$ where $M_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and $M_1 = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$. The authors have proved that in the case when the heights of the variables are greater than 1, each filtered Lie algebra associated with a graded non-alternating Hamiltonian algebra is given by a symmetric Hamiltonian differential form with non-constant coefficients. It has been proven that such forms can be reduced to Hamiltonian forms with constant coefficients by an admissible change of variables. Thus, in the case when variables are of large enough heights the graded nonalternating Hamiltonian Lie algebras are rigid with respect to filtered deformations.

Note that invariants of skew-symmetric forms were found in [10]. The case when the hight of variables may be equal to 1 is more complicated. In [11] were found isomorphisms between known simple 14-dimensional Lie algebras of characteristic 2. In particular, an isomorphism between $P(4 : \mathbf{1})$ and classical Hamiltonian Lie algebra $H(4, \mathbf{1}, \omega)$ was constructed.

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References

[1] 1. Shalev A. *The structure of finite p-groups: effective proof of the coclass conjectures.* Invent. Math., 1994, V. 115, P. 315-345.

[2] 2. Shalev A. Zelmanov E.I. *Pro-p groups of finite coclass.* Math. Proc. Cambridge Philos. Soc., 1992, V. 111, P. 417-421.

[3] 3. Benkart G., Kostrikin A.I., Kuznetsov M.I. *Finite-dimensional simple Lie algebras with a nonsingular derivation.* J. Alg, 1995, V. 171, P. 894-916.

[4] 4. Kostrikin A.I., Shafarevich I. R. *Graded Lie algebras of finite characteristic.* Izv. AN SSSR. Ser. Matem., 1969, V.33(2), P. 251-322. (Russian).

[5] 5. Lin L. *Non-alternating Hamiltonian algebra P(n, m) of characteristic 2.* Comm. Alg., 1993, V.21, P. 399-411.

[6] 6. Bouarrouj S., Grozman P., Lebedev A., Leites D. *Divided power (co)homology. Presentation of simple finite dimensional modular superalgebras with Cartan Matrix.* Homology, Homotopy Appl., 2010, V.12(1), P. 237-248.

[7] 7. Bouarroudj S., Grozman P., Lebedev A., Leites D., Shchepochkina I. *New simple Lie algebras in characteristic 2.* Math. Research Notices., 2009, V.2015, P. 1-16.

[8] 8. Yier U., Leites D., Messaoudene M., Shchepochkina I. *Examples of simple vectorial Lie algebras in characteristic 2.* J. Nonlin. Math. Phys., 2010, V.17, Suppl. 1, P. 311-374.

[9] 9. Kuznetsov M.I., Kondrateva A.V., Chebochko N.G. *On Hamiltonian Lie algebras of characteristic 2.* Matem. J. (NAS of Kazakhstan)., 2016, V.16, N.2, P. 54-65. (Russian).

[10] 10. Skryabin S. *Classification of Hamiltonian forms over divided power algebras.* Matem. Sb., 1990, V.181(1), P. 114-133. (Russian).

[11] 11. Kuznetsov M.I., Kondrateva A.V., Chebochko N.G. *Simple 14-dimensional Lie algebras in characteristic 2.* Zapiski Nauch. Sem. POMI., 2017, V.460, P. 158-167. (Russian).

[12] 12. Chebochko N.G., Kuznetsov M.I. *Integrable cocycles and global deformations of Lie algebra of type G_2 in characteristic 2.* Commun. Alg., 2017, V.45(7), P. 2969-2977.

Homoclinic Bifurcation in Morse-Novikov Theory

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Morse-Novikov Theory deals with a closed manifold M endowed with a Morse function which is only defined up to an additive constant (that is, a closed differential form of degree one). The gradient X (relative to some auxiliary Riemannian metric) is well defined. Generically, X is a Kupka-Smale vector field, meaning that it has no homoclinic orbit. In such a case, S. Novikov (1981) has defined a complex encoding the orbits connecting zeroes of consecutive Morse index; possibly, there are infinitely many of them!

But, homoclinic connecting orbits may appear in a one-parameter family of gradients. We analyze carefully such bifurcations and their dramatic effects on the Morse-Novikov complex. An unexpected doubling phenommenon may happen.

Realization of Manifolds as Leaves of Compact Foliated Spaces Using Graph Colorings

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Many (quasi-isometric types of) manifolds were shown to be non-realizable as leaves of foliations on compact manifolds. Results of this type were achieved by Phillips and Sullivan, Januszkiewicz, Cantwell and Conlon, Cass, Schweitzer, Attie and Hurder, Zeghib, and Meniño Cotón and Schweitzer. Despite of the existence of such"non-leaves" for compact foliated manifolds, we prove that any (repetitive) Riemannian manifold of bounded geometry can be realized as a leaf of a (minimal) compact foliated space without holonomy. The difference is that the local transversals to the leaves can be arbitrary Polish spaces in the case of foliated spaces, whereas the local transversals have to be manifolds in the case of foliations. The main tool of the proof is a theorem with independent interest that we have also shown. It states that, if an infinite (repetitive) connected graph X has an upper bound Δ on the vertex degrees, then X has a (repetitive) limit aperiodic vertex coloring by Δ colors. This is a joint work with Ramón Barral Lijó.

Arboreal Cantor Actions

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The asymptotic discriminant is an invariant of actions of discrete groups on Cantor sets, recently introduced by the speaker in a joint work with Hurder. The asymptotic discriminant arises as a sequence of surjective group homomorphisms of certain profinite groups, associated to the action.

An arboreal representation of the absolute Galois group of a field is a profinite group, acting on the boundary of a spherically homogeneous rooted tree. In this talk, we show how one can compute the asymptotic discriminant for such representations. We give examples of arboreal representations with stable and wild asymptotic discriminant.

The talk is based on the results of the article

O. Lukina, *Arboreal Cantor actions*, https://arxiv.org/abs/1801.01440.

Diffeomorphisms Preserving Morse-Bott Functions

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Let M be a smooth compact manifold and P be either the real line or the circle. Notice also that there is a natural right action $\nu : \mathcal{C}^\infty(M,P) \times \mathcal{D}(M) \to \mathcal{C}^\infty(M,P)$, defined by $\nu(f,h) = f \circ h$ of the groups of diffemorphisms $\mathcal{D}(M)$ of M on the space $\mathcal{C}^\infty(M,P)$ of smooth maps $M \to P$. For $f \in \mathcal{C}^\infty(M,P)$ and a subset $X \subset M$ let

$$S(f) = \{h \in \mathcal{D}(M) \mid f \circ h = f\}, \qquad S(f,X) = S(f) \cap \mathcal{D}(M,X)$$

be the stabilizers of f with respect to the above action of $\mathcal{D}(M)$ and the induced action of $\mathcal{D}(M,X)$. Let also $S_{id}(f)$ and $S_{id}(f,X)$ be the identity path components of the corresponding stabilizers.

Theorem. *Let $f : M \to P$ be a Morse-Bott map of smooth compact manifold M, so the set Σ_f of critical points of f is a disjoint union of smooth mutually disjoint closed submanifolds $C_1, \ldots, C_k$. Let also $X \subset M \setminus \Sigma_f$ be a closed (possibly empty) subset. Then the maps*

$$\rho\colon S(f,X) \to \mathcal{D}(\Sigma_f), \qquad \rho(h) = h|_{\Sigma_f},$$

$$\rho_0\colon S_{id}(f,X) \to \mathcal{D}_{id}(\Sigma_f) \equiv \prod_{i=1}^{k} \mathcal{D}_{id}(C_i), \qquad \rho_0(h) = (h|_{C_1}, \ldots, h|_{C_k}),$$

are locally trivial fibrations over their images, and the map ρ_0 is surjective.

This result can be regarded as a variant of the well know result Cerf and Palais on local triviality of restrictions to critical submanifolds of Morse-Bott function f for f-preserving diffeomorphisms.

References

[1] Olexandra Khohliyk, Sergiy Maksymenko, *Diffeomorphisms preserving Morse-Bott functions*, `arXiv:1808.03582`

Non-monotonicity of the Kneading Invariant for Lorenz Maps with Infinite Derivatives

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The problem on monotonicity of the kneading invariant (and, as a corollary, of the topological entropy) for one-parameter families of unimodal maps was ispired by J.Milnor and W.Thurston in 70s of the last century. Milnor proved monotonicity of the kneading invariant for the classical quadratic family $f_c(x) = x^2 + c$, and since that time there had been several results on monotonicity for different families of smooth maps. A recent result by Van Strien, Levin and Shen states the monotonicity of the kneading invariant for families of the form $f^d(x) = |x|^d + c$ for sufficiently large d.

Our aim is to study the dependence of the kneading for symmetric Lorenz maps $f_{c,\varepsilon}(x) = (-1 + c|x|^{1-\varepsilon})sign(x)$. This is the normal form for splitting the homoclinic loop in systems that have a saddle equilibrium with one-dimensional unstable manifold provided that the saddle value is negative. L.P.Shilnikov proved that such a bifurcation corresponds to the birth of Lorenz attractor.

In contrast to the maps considered above, this family has an infinite derivative at the point of discontinuity, which leads to a new phenomenon. Numerical experiments show that the kneading invariant does not change monotonically with respect to the parameter c. In the special parameters domain, there exists an interval $(0 < \varepsilon < \varepsilon *)$, in which the kneading invariant has a single minimum as a function of the parameter c.

In the talk we discuss bifurcations that appear in this family and we indicate some domains in the bifurcation plane where the kneading invariant depends monotonically.

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Holonomy Pseudogroup of a Manifold over the Algebra of Dual Numbers and its Applications

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The algebra of dual numbers $\mathbb{D} = \mathbb{R}(\varepsilon)$ is a 2-dimensional $\mathbb{R}$-algebra with basis $\{1, \varepsilon\}$ and multiplication defined by the relation $\varepsilon^2 = 1$. An arbitrary element of $\mathbb{D}$ has the form $X = a + b\,\varepsilon$, where $a, b \in \mathbb{R}$. Denote by $\mathbb{D}^n$ the $\mathbb{D}$-module of n-tuples $\{X^1, \ldots, X^n\}$, $X^i = x^i + \dot{x}^i\varepsilon$, $i = 1, \ldots, n$, of dual numbers. The equations $x^i = x^i_0 = const$, $i = 1, \ldots, n$, define the *canonical foliation* $\mathcal{F}$ on $\mathbb{D}^n$.

Let $U \subset \mathbb{D}^n$ be an open subset. A mapping $F : U \to \mathbb{D}^m$ is called $\mathbb{D}$-*smooth* if the tangent mapping T_xF is $\mathbb{D}$-linear for all $x \in U$. The structure of n-dimensional $\mathbb{D}$-smooth manifold on a real $2n$-dimensional manifold M_{2n} is given by an atlas with charts taking values in $\mathbb{D}^n$ and transition functions being $\mathbb{D}$-smooth diffeomorphisms. A $\mathbb{D}$-smooth manifold $M_n^{\mathbb{D}}$ is called *complete* if the leaves of its canonical foliation are complete affine manifolds. For a $\mathbb{D}$-smooth manifold $M_n^{\mathbb{D}}$ and a complete immersed transversal $\varphi : W_n \to M_n^{\mathbb{D}}$, we define the *holonomy pseudogroup* Γ_W. We apply holonomy pseudogroups to the study of $\mathbb{D}$-diffeomorphisms between complete $\mathbb{D}$-smooth manifolds.

Theorem 1. 1) *Let $M_n^{\mathbb{D}}$ be a complete $\mathbb{D}$-smooth manifold, and let $\varphi : W_n \to M_n^{\mathbb{D}}$ be an immersion of a complete transversal. Then TW_n/Γ_W is a $\mathbb{D}$-smooth manifold $\mathbb{D}$-diffeomorphic to $M_n^{\mathbb{D}}$.* 2) *If holonomy pseudogroups of two complete $\mathbb{D}$-smooth manifolds on immersed complete transversals are isomorphic, then these manifolds are $\mathbb{D}$-diffeomorphic.*

Theorem 2. *Let $F : M_n^{1\mathbb{D}} \to M_n^{2\mathbb{D}}$ be a diffeomorphism between two $\mathbb{D}$-smooth manifolds which is a foliated isomorphism with respect to the canonical foliations, and let $\varphi : W_n \to M_n^{1\mathbb{D}}$ be an immersion of a complete transversal. The mapping F is a $\mathbb{D}$-diffeomorphism if and only if the holonomy pseudogroups on the immersed complete transversals φ and $F \circ \varphi$ coincide.*

The quotient $\mathbb{D}/\Lambda$ of the algebra $\mathbb{D}$ by a lattice Λ carries a structure of $\mathbb{D}$-smooth manifold called a $\mathbb{D}$-torus and denoted by $T(\Lambda)$.

Theorem 3. *$\mathbb{D}$-tori $T(\Lambda)$ and $T(\Lambda')$ are $\mathbb{D}$-diffeomorphic if and only if there exists an element $A = a + \dot{a}\varepsilon \in \mathbb{D}$ such that $\Lambda' = A\Lambda$. In the case when leaves of the canonical foliations on two $\mathbb{D}$-diffeomorphic $\mathbb{D}$-tori $T(\Lambda)$ and $T(\Lambda')$ are everywhere dense, a $\mathbb{D}$-diffeomorphism between $T(\Lambda)$ and $T(\Lambda')$ is given by the formula $\widetilde{F}(X) = AX + B$, where $A, B \in \mathbb{D}$ and the element A is uniquely determined.*

Let M_n be an affine manifold [1] with atlas $\{(U_\alpha, h_\alpha)\}$, $h_\alpha : U_\alpha \ni x \mapsto \{x^i_\alpha\} \in \mathbb{R}^n$ and transition functions of the form $x^i_\alpha = a^i_k(\alpha, \beta)x^k_\beta + b^i(\alpha, \beta)$. With M_n one can naturally associate two locally trivial bundles $\pi : OM_n \to M_n$ and $\tilde{\pi} : \tilde{O}M_n \to M_n$ [1], [2] whose transition functions have the form $\{x^i_\alpha = a^i_k(\alpha, \beta)x^k_\beta + b^i(\alpha, \beta),\ y^i_\alpha = a^i_k(\alpha, \beta)y^k_\beta + b^i(\alpha, \beta)\}$ and $\{x^i_\alpha = a^i_k(\alpha, \beta)x^k_\beta + b^i(\alpha, \beta),\ y^i_\alpha = a^i_k(\alpha, \beta)y^k_\beta\}$ respectively.

Assigning to an element X with coordinates $\{x^i, y^i\}$ from $\pi^{-1}(U_\alpha)$ (respectively, from $\tilde{\pi}^{-1}(U_\alpha)$) the element $\{y^i_\alpha + \varepsilon x^i_\alpha\} \in \mathbb{D}^n$, we introduce on OM_n (respectively, $\tilde{O}M_n$) a structure of $\mathbb{D}$-smooth manifold [2]. Denote the obtained $\mathbb{D}$-smooth manifolds, respectively, by $O^{\mathbb{D}}M_n$ and $\tilde{O}^{\mathbb{D}}M_n$.

An affine manifold M_n is called *radiant* [1] if it has an atlas with linear transition functions.

Theorem 4. *$\mathbb{D}$-smooth manifolds $O^{\mathbb{D}}M_n$ and $\tilde{O}^{\mathbb{D}}M_n$ are $\mathbb{D}$-diffeomorphic if and only if M_n is a radiant affine manifold.*

References

[1] Goldman W., Hirsch M. W. *The radiance obstruction and parallel forms on affine manifolds.* Trans. Amer. Math. Soc. **286** (2), 1984, 629–649.

[2] Malyugina A. A., Shurygin V. V. *Holonomy representation for a class of manifolds over the algebra of dual numbers.* Izv. Penz. gos. pedagog. univ. im. V. G. Belinskogo. 2011, no. 26, 128–136.

A Parameter Space of Cubic Newton Maps With Parabolics

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Abstract. A parameter space of cubic rational Newton maps for entire maps that take the form $(z^2 + a)e^z$ parametrized by a single complex parameter $a \neq 0$ is studied. The topological properties of stable components, the components of cubic Newton maps for which the free critical point belongs to attracting basins and the basin of the parabolic fixed point at ∞, are obtained. When the free critical point belongs to the basin of ∞, the quasiconformal conjugacy classes in a corresponding stable component were explicitly constructed. It is proved that each stable component contains a unique center, conformally rigid f_a, which is a postcritically minimal Newton map, moreover, each quasiconformal conjugacy class comes from conformally rigid different types of f_a, that we introduce them as postcritically non-minimal Newton maps.

Introduction. The Newton map of an entire function $g : \mathbb{C} \to \mathbb{C}$ is the meromorphic map N_g defined by $N_g(z) := z - g(z)/g'(z)$. The Newton maps for the family of entire functions $g(z) = (z^2 + a)e^z$ parametrized by a complex number $a \neq 0$, is the family given by the cubic rational maps $f_a(z) = z - \frac{z^2+a}{z^2+2z+a}$. The fixed points of f_a are the roots of $z^2 + a = 0$, which are superattracting, and a point at infinity is parabolic of multiplier $+1$ with one attracting petal, thus its Julia set is connected. We have a single "free" critical point that determines dynamical properties of the map.

Critical orbit relations. Let c_1 and c_2 be critical points of a rational function f. We say that c_1 and c_2 are in a critical orbit relations if $f^{\circ m}(c_1) = f^{\circ n}(c_2)$ for some non-negative integers m and n, if $c_1 = c_2$ we require $m \neq n$.

Lemma 1. *All possible relations are of the form $f_a^{\circ n}(c_2) = c_1$ for some integer $n \geq 0$.*

This leads us to introduce the following new notions:

Definition 1 (Minimal and non-minimal critical orbit relations)**.** *Let f_a be a cubic Newton map defined above and let $c_1 \in U_1$ and $c_2 \in U_2$ be its critical points with U_1 and U_2 the connected components of the basin of the parabolic fixed point at ∞. Assume $f_a^{\circ m}(U_1) = U_2$ with minimal such $m \geq 0$. We say that c_1 and c_2 are in* minimal critical orbit relations *if $f_a^{\circ m}(c_1) = c_2$ with the same m. If $f_a^{\circ n}(c_1) = c_2$ with $n > m$ then we say that c_1 and c_2 are in* non-minimal critical orbit relations.

Definition 2 (Postcritically minimal and Postcritically non-minimal cubic Newton maps). *A cubic Newton map f_a is called postcritically minimal (postcritically non-minimal) if its Fatou set consists of superattracting basins and the parabolic basin of ∞ and if its free critical point*

(a) has a finite orbit when it is on the Julia set or in superattracting basins; or

(b) is capture by another critical point on the parabolic basin and form minimal critical orbit relations (non-minimal critical orbit relations) such that if c is a critical point in a component U such that $f_a^{\circ m}(U)$ is an immediate basin of ∞ with minimal such $m \geq 0$ then $f_a^{\circ m}(c)$ is a critical point with the same m (then $f_a^{\circ n}(c)$ is a critical point with $n > m$).

Stable components in the parameter space. The components of cubic Newton maps for which the free critical point belongs to attracting basins and the basin of the parabolic fixed point at ∞ are called stable components. Our main result is the following.

Theorem 1. *Every stable component is a topological 2-cell and contains its center, which is a cubic postcritically minimal Newton map. When the free critical point belongs to the basin of ∞, the quasiconformal conjugacy classes in a corresponding stable component is a quasiconformal deformation of a unique stable cubic postcritically non-minimal Newton map. Moreover, each of these classes is a half-strip. The stable components corresponding to the parabolic fixed point splits into such half-strips.*

Upper Bounds of Morse Numbers of the Matrix Elements Irreducible Representations of Simple Compact Connected Lie Groups

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Based on the differential-topological approach to the analysis of the properties of the matrix elements of real irreducible representations of connected compact simple Lie groups G previously we have classified their so-called taut representations [1, 2].

It turned out that among non-commutative groups only compact Lie groups $O(n)$, $U(n)$, and $Sp(n)$ in their standard representations are taut representations and the minimum number of critical points of Morse matrix elements is the total Betty number of the indicated groups G. The Morse numbers of the matrix elements of the other real irreducible representations $\rho : G \to Aut(R^N)$ are strictly greater than the total Betty number of G. In the theorem stated below, we indicate the upper bounds on the minimum number of critical points of Morse matrix elements from the space of matrix elements $M(\rho)$ of the representation ρ in terms of the highest weight λ of the representation and the geometric characteristics of the Lie group G, based on the integral-geometric formulas of [3, 4].

It is important that all functions from the space of matrix elements $M(\rho_\lambda$ of the representation ρ_λ with the highest weight λ are the eigenfunctions of the bi-invariant Laplace operator on G, belonging to the eigenvalue $E_\lambda =< \lambda+\delta, \lambda+\delta > - < \delta, \delta >$, where δ half-sum of positive roots of a Lie algebra of a group G and $<,>$ is a Cartan–Killing metric.

Theorem. *The minimum number $\gamma(\rho_\lambda)$ of the critical points of the Morse matrix elements from the space of matrix elements $M(\rho_\lambda)$ of the real irreducible representation $\rho_\lambda : G \to Aut(R^N)$ of a* *connected compact simple group Lie G with the highest weight λ is estimated above by the number $2/\sigma_n(E_\lambda/n)^{n/2}vol(G)$, where σ_n is the volume of the n-dimensional sphere of radius $1, n = dimG, E_\lambda$ is the eigenvalue Laplace operator corresponding to the highest weight λ and $vol(G)$ is the volume of the Lie group G with respect to the Riemannian element of the Cartan–Killing metric.*

Remarks.

1. The upper bound indicated in the theorem is exact and is achieved on groups $SU(2)$ or $SO(3)$ in representations of the minimum dimension.

2. In December 2017, on the conference dedicated to 80th anniversary of V.I. Arnold, K. Kozhasov [5] announced the following result concerning an exact upper bound the number of critical points $C_{d,n}(f)$ of harmonic polynomials f on the sphere S^{n-1}, corresponding to the eigenvalue - $d(d+n-2)$ of the spherical Laplace operator:

$$C_{d,n}(f) \leq 2[(d-1)^{n-1} + (d-1)^{n-2} + ... + (d-1) + 1].$$

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References

[1] C. Gorodski, *Taut representation of compact simple Lie group.* Illinois J. Math. 52 (2008), no. 1, pp. 121-143

[2] M.V. Meshcheryakov, *Classification of the taut linear irreducible representations of compact connected Lie groups.* The fourth school–conference "Lie algebras, algebraic groups and the theory of invariants" Moscow, Russia. January 27 - February 1, 2014. Abstracts.

[3] V.M. Gichev, *Metric properties on compact isotropy irreducible homogeneous spaces.* Anal. Math Phys. 3, no. 2 (2013), pp. 119-144.

[4] D. Akhiezer, B. Kazarnovskii. *On common zeros of eigenfunctions of the Laplace operator.* Abh. Math. Sem. Univ. Hamburg. 87, no. 1 (2017) pp. 105-111.

[5] K. Kozhasov. *On spherical harmonics with maximum number of critical points.* International Conference "Contemporary Mathematics". Moscow, Russia. December 18-23, 2017.

Realization of Diffeomorphisms with One Saddle Orbit

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In the study of discrete dynamical systems, i.e., the study of orbits of self-maps defined on a given compact manifold, the periodic behavior plays an important role. In the last forty years there was a growing number of results showing that certain simple hypotheses force qualitative and quantitative properties (like the set of periods) of a system. One of the best-known results is the title of the paper “Period three implies chaos for the interval continuous self-maps” [5]. The effect described there was discovered by O. M. Sharkovsky in [7]. The most useful tools for proving the existence of fixed points or, more generally, of periodic points for a continuous self-map f of a compact manifold is the Lefschetz fixed point theorem and its improvements (see, for instance [2] and [3]). The Lefschetz zeta-function simplifies the study of the periodic points of f. This is a generating function for all the Lefschetz numbers of all iterates of f. The periodic data of diffeomorphisms with regular dynamics on surfaces were studied by means zeta-function in a series of already classical works by such authors as Paul R. Blanchard, John M. Franks, Rufus Bowen, Steve Batterson, John Smillie, William H. Jaco, Peter B. Shalen, Carolyn C. Narasimhan, and others. A description of periodic data of gradient-like diffeomorphisms of surfaces was given in [1] by means of classification of periodic surface transformations obtained by Jakob Nielsen [6]. In [4], the authors show that the study of periodic data of arbitrary Morse-Smale diffeomorphisms on surfaces is reduced by filtration to the problem of computing periodic data of diffeomorphisms with a unique saddle periodic orbit. Polar diffeomorphisms of the surface are considered, that is, iffeomorphisms having a single sink and a single source periodic orbit. A classic example of such a diffeomorphism is the sink-source diffeomorphism, which has no saddle points and exists only on a two-dimensional sphere. However, the addition of even a single saddle orbit greatly expands the class of polar diffeomorphisms on surfaces. All possible types of periodic data for such polar diffeomorphisms have been established, and it is shown that the saddle orbit always has a negative orientation type. It is proved that every orientable surface admits a Morse-Smale diffeomorphism with one saddle orbit that preserves orientation. It is shown that these diffeomorphisms have exactly three nodal orbits. In addition, all possible types of periodic data for such diffeomorphisms are established. The report presents the realization of diffeomorphisms with one saddle orbit on surfaces of any genus depending on the periodic data.

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References

[1] A. N. Bezdenezhnykh and V. Z. Grines, Realization of gradient-like diffeomorphisms of twodimensional manifolds (Russian) in Differential and Integral Equations (Russian). Ed. N. F.Otrokov. Gorki: Gorkovskii Gos. Univ., 1985. 33 – 37, 124 – 125. Trans. Sel. Math. Sov. 11 (1985), no. 1, 19 – 23.

[2] Robert F. Brown, The Lefschetz Fixed Point Theorem. Glenview, Ill.-London: Scott, Foresman and Co., 1971.

[3] John M. Franks, Homology and Dynamical Systems. CBMS Regional Conference Series in Mathematics, 49. Providence, R. I.: American Mathematical Society, 1982.

[4] V. Z. Grines, O. V. Pochinka, and S. van Strien, A complete topological classification of Morese-Smale diffeomorphisms on surfaces: A kind of kneading theory in dimension two.

[5] T. Y. Li and James A. Yorke, Period three implies chaos, Amer. Math. Monthly 82 (1975), no. 10, 985 – 992. Available at arXiv:1712.02230 [Math DS] December 6, 2017.

[6] Jakob Nielsen, Die Struktur periodischer Transformationen von Flächen. Phys. Meddelerser 15, no. 1, (1937), 1-77.

[7] O. M. Sharkovsky, Co-existence of cycles of a continuous mapping of the line into itself (Russian), Ukrain. Mat. Z. 16 (1964), 61 – 71.

On Minimal Non-invertible Maps

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In the late 1960s J. Auslander asked questions concerning the existence of minimal non-invertible maps. Since then many examples of such maps have become available, but many questions remain as to which spaces admit such maps.

In 2003, Bruin, Kolyada and Snoha asked whether the circle is the only infinite continuum that admits a minimal homeomorphism but no non-invertible minimal map. They also suspected that R.H. Bing's pseudo-circle might provide an answer for the above question.

In 2017, the question was answered in the negative by Downarowicz, Snoha and Tywoniuk by the construction of a family of continua which admit minimal homeomorphisms and do not admit minimal non-invertible maps.

Recently in joint work with Boroński, Liu and Kennedy we proved that pseudo-circle does not serve as another counterexample, by proving that it admits a noninvertible minimal map.

In this talk we will present a few results and techniques related to the above question and answers mentioned above.

Sets of Group Actions with Various Shadowing Properties

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In this talk, we discuss various shadowing properties of classical dynamical systems (actions of $\mathbf{Z}$) as well as of actions of more general finitely generated groups.

A Buffer Phenomenon in a System of Two Non-Linearly Coupled Relaxation Oscillators

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We consider a mathematical model of synaptic delay coupled neuron type oscillators

$$\begin{aligned} \dot{u}_1 &= [\lambda f(u_1(t-1)) + b\, g(u_2(t-h))\ln(u_*/u_1)]u_1, \\ \dot{u}_2 &= [\lambda f(u_2(t-1)) + b\, g(u_1(t-h))\ln(u_*/u_2)]u_2. \end{aligned} \tag{1}$$

This model uses an idea of FTM (Fast Threshold Modulation) and is based on a model suggested in [1]. A difference is an coupling delay $h > 1$.

Here, $u_1(t), u_2(t) > 0$ are normalized membrane potentials of neurons. A parameter $\lambda \gg 1$ is large and characterizes the rate of electric processes in the system. Parameter $b = const > 0$,

$u_* = \exp(c\lambda)$ is a threshold value, where $c = const \in R$. Functions $f(u), g(u) \in C^2(R_+)$ satisfy the following conditions:

$$f(0) = 1,\ g(0) = 0;\ \forall u > 0\ g(u) > 0; a > 0;\ \text{as } u \to +\infty$$
$$f(u) + a, g(u) - 1, uf'(u), u^2 f''(u), ug'(u), u^2 g''(u) = O\big(u^{-1}\big).$$

We make substitution $u_j = \exp(\lambda x_j)$, $j = 1, 2$. Then, as $\lambda \to +\infty$, we get the following limit object:

$$\begin{aligned} \dot{x}_1 &= R(x_1(t-1)) + b\,(c - x_1)H(x_2(t-h)), \\ \dot{x}_2 &= R(x_2(t-1)) + b\,(c - x_2)H(x_1(t-h)), \end{aligned} \tag{2}$$

where

$$R(x) \stackrel{\text{def}}{=} \begin{cases} 1, & \text{whenever } x \le 0, \\ -a, & \text{whenever } x > 0, \end{cases} \qquad H(x) \stackrel{\text{def}}{=} \begin{cases} 0, & \text{whenever } x \le 0, \\ 1, & \text{whenever } x > 0. \end{cases}$$

It is relay system. We prove that there is a *buffer phenomenon* for system (2).

Definition 1. *We speak of the buffering phenomenon if the system under consideration can have an arbitrary fixed number of stable periodic modes if the system parameters are properly chosen.*

We prove the following.

Claim. *There exists a domain in a parameter space of system (2) such that for any $n \in N$ there exists h such that there exist $n-1$ stable periodic solutions in a phase space of (2). Components $x_1(t)$ and $x_2(t)$ of the solutions have, respectively, m and $n-m$ $(m = 1, \ldots, n-1)$ relatively short alternating segments of positivity and negativity which go after a long enough segment where the solution values are negative.*

For system (1), a numerical modelling with correspondence initial condition detects a buffer phenomenon.

Since u_i depends on x_i $(i = 1, 2)$ exponentially, solution properties described in Claim means that $u_1(t)$ and $u_2(t)$ have m and $n-m$ impulses, respectively. Such behaviour is called *bursting-effect* in a neurodynamics.

Definition 2. *A bursting-effect is an alternation of several consecutive intensive spikes with refractory period of a membrane potential.*

Note that the obtained effects (buffering and bursting) are due to the delay in the connection chain.

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References

[1] Glyzin S. D., Kolesov A. Yu., Rozov N. K. On a method for mathematical modeling of chemical synapses // Differential Equations. 2013. V. 49. № 10. P. 1193-1210.

On Two-parameter Perturbations of the Harmonic Oscllator Equation

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We consider families $\xi = \{X_\mu\}$ of equations

$$X_\mu : \ddot{x} + x = \mu_1 f_1(x, \dot{x}, \mu) + \mu_2 f_2(x, \dot{x}, \mu),$$

where $\mu = (\mu_1, \mu_2)$, f_k are C^5-functions on $G \times [-\sigma, \sigma]^2$, $G = \{(x, y) \colon x^2 + y^2 \le \bar{R}^2\}$, such that $f_k(0, 0, \mu) = 0, k = 1, 2,$ and corresponding autonomous systems

$$\dot{x} = y, \quad \dot{y} = -x + \mu_1 f_1(x, \dot{x}, \mu) + \mu_2 f_2(x, y, \mu).$$

The set $\mathcal{F}$ of all such families is identified with linear space of C^5-functions $(f_1, f_2) \colon G \times [-\sigma, \sigma]^2 \to \mathbf{R}^2$ with norm

$$\|\xi\| = \max_{k=1,2} \max_{(x,y,\mu)} \max_{0 \le i+j \le 3} |\partial^{i+j} f_k(x, y, \mu)/\partial x^i \partial y^j|.$$

Let

$$J_k(u) = \int_0^{2\pi} f_k(u \cos\varphi, u \sin\varphi, 0) \sin\varphi \, d\varphi, \quad k = 1, 2; \quad J(u) = (J_1(u), J_2(u));$$

$$D(u) = J_1(u) J_2'(u) - J_1'(u) J_2(u).$$

It is clear that $D(0) = D'(0) = D''(0) = 0$.

Let $\mathcal{SF}$ be the subset of $\mathcal{F}$ defined by conditions: 1) $D(u)$ has on $(0, R), 0 < R < \bar{R}$, only simple zeroes; 2) $D(R) \ne 0$; 3) $D'''(0) \ne 0$; 4) if $D(u_1) = D(u_2) = 0$, $0 < u_1 < u_2 < R$, then vectors $J(u_1)$ and $J(u_2)$ are linear independent; 5) if $D(u) = 0$, $0 < u < R$, then vectors $J(u)$ and $J(R)$, $J(u)$ and $J'(0)$ are linear independent.

Theorem 1. *The set $\mathcal{SF}$ is open and everywhere dense in $\mathcal{F}$.*

Denote by T the arc $[0, R] \times \{0\} \subset G$. Let $\|\cdot\|$ be the Euclidean norm in $\mathbf{R}^2$.

Theorem 2. *Let for family $\{X_\mu\} \in \mathcal{SF}$ $0 = u_0 < u_1 < \cdots < u_n$ be the zeroes of function $D(u)$, and $u_{n+1} = R$. Let $q_0, q_1, \ldots, q_{n+1}$ be a permutation of integers $0, 1, \ldots, n+1$, such that numbers $\theta_i \in [0, \pi)$, where $\cos\theta_i = -sgn J_1(u_{q_i}) J_2(u_{q_i})/\|J(u_{q_i})\|$, $\sin\theta_i = sgn J_1(u_{q_i}) J_1(u_{q_i})/\|J(u_{q_i}\|$ if $J_1(u_{q_i}) \ne 0$, and $\theta_i = 0$ if $J_1(u_{q_i}) = 0$, for the case $u_{q_i} \ne 0$; $\cos\theta_i = -sgn J_1'(0) J_2'(0)/\|J'(0)\|$, $\sin\theta_i = sgn J_1'(0) J_1'(0)/\|J'(0)\|$ if $J_1'(0) \ne 0$, for the case $u_{q_i} = 0$, are in the following order: $0 \le \theta_0 < \cdots < \theta_n < \theta_{n+1} < \pi$.*

Then for some $\delta > 0$ there is a partition of the set $M = \{\mu = (\mu_1, \mu_2) : 0 < \mu_1^2 + \mu_2^2 < \delta^2\}$ into subsets M_i^+, M_i^-, B_i^+ and B_i^- $(i = 0, 1, \ldots n+1)$ such that in coordinates (θ, ρ), $x = \rho\cos\theta$, $y = \rho\sin\theta$ in M $B_i^\pm = \{\mu \in M \colon \theta = \theta_i^\pm(\rho)\}$ for $i = 0, \ldots n$, $M_i^\pm = \{\mu \in M \colon \theta_{i-1}^\pm(\rho) < \theta < \theta_i^\pm(\rho)\}$ for $i = 1, \ldots n+1$, $M_0^+ = \{\mu \in M \colon \theta_{n+1}^-(\rho) - \pi < \theta < \theta_0^+(\rho)\}$, $M_0^- = \{\mu \in M \colon \theta_{n+1}^+(\rho) < \theta < \theta_0^-(\rho)\}$, where $\theta_j^+ : [0, \delta) \to \mathbf{R}$ $(\theta_j^- : [0, \delta) \to \mathbf{R})$ is a C^1- function, $\theta_j^+(0) = \theta_j$ $(\theta_j^-(0) = \theta_j + \pi)$;

1) an equation X_μ, $\mu \in M_i^\pm$, $(i = 0, \ldots n+1)$ has a hyperbolic singular point - the focus $(0, 0)$ and only hyperbolic closed orbits intersecting the arc T at interior points;

2) an equation X_μ, $\mu \in B_i^\pm$, $(q_i \ne 0, n+1)$ has a hyperbolic singular point - the focus $(0, 0)$, all its closed orbits intersect the arc T at interior points, one of them is a double cycle, and the rest are hyperbolic limit cycles;

3) an equation X_μ, $\mu \in B_i^{\pm}$, ($q_i = 0$) has the weak focus $(0, 0)$ *and only hyperbolic closed orbits intersecting the arc T at interior points;*

4) an equation X_μ, $\mu \in B_i^{\pm}$, ($q_i = n + 1$) has a hyperbolic singular point - the focus $(0, 0)$ *and only hyperbolic closed orbits intersecting the arc T , one of them at the point* $(R, 0)$.

An Estimates for a Surface of Given Topological Type with Given Mean Curvature

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This paper describes a surface in R^3 with a mean curvature that equals value of some function in each point. Equation wich represents surfaces of given mean curvature is obtained. Conditions wich enable it to be resolvable is obtained.

Let S be a compact regular surface locally defined by equation $r = r^S(u, v)$ in R^3. Let a fuction $H(x, y, z)$ be given in some locality of S. We consider a question of the existance of some surface S^f homeomorphic to S, defined by an equation $r = r^S(u, v) + f(u, v)\overline{n}^S(u, v)$ and in each point A has avirage curviture $H(A)$. This problem has been considered in [1] (271-303) and in [2] for the case when S is a sphere or a torus.

There is a coordinate system (u, v, ρ) emerges in the locality of S, where (u, v) - local coordinates on S and ρ is offset along perpendicular to S. This problem reduces to the question of some second-order differential equation solvability on $f(u, v)$ within S . Evaluation of solution and of first derivatives of solution is required for the proof of solvability of this equation.

Let S^ρ be a surface defined by an equation $r = r^S(u, v) + \rho\overline{n}^S(u, v)$ where ρ is a constant, such that $|\rho| < c$. Here $c = min_{(A \in S, i=\overline{1,2})}\{\frac{1}{k_i(A)}\}$ and $k_i(A)$ are main normal curvitures of S at the point A. Mean curviture of S^ρ equals $H^\rho = \frac{k_1}{1-\rho k_1} + \frac{k_2}{1-\rho k_2}$.

We represent H as a sum

$$H(u, v, \rho) = H^\rho(u, v, \rho) + h(u, v, \rho).$$

Theorem. *If a and b are constants, and $-c < a < b < c$, and if*

$$h(u, v, \rho) < 0 \ \textit{when} \ \rho < a,$$

$$h(u, v, \rho) > 0 \ \textit{when} \ \rho > b,$$

there are following estimates hold for the function $f(u, v)$:

$$a < f(u, v) < b.$$

References

[1] Bakelman I., Verner A., Kantor B. *Introduction in differential geometry at all.* –Science, 1973. – 440 c.

[2] Golubcova T. *Estimates of torus homeomorphic surface in E^3 with given average curviture* // Geometry and topology. –1974. – № 2. – c. 76–88.

Limit Properties of Independent Random Semigroups Compositions

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The notion of random operator valued functions and random group of bounded linear operators in Hilbert space will be investigated (see [1]). The topology on the vector space of operator valued functions is introduced. The distributions on the topological vector space of operator valued functions corresponding to the sequence of compositions of random functions are studied.

The Feynman iteration of random operator valued function will be introduced as the sequence of compositions of independent identically distributed random operator valued functions. The convergence of the mean values of Feynman iteration of random 1-parametric semigroup to some averaged semigroup is obtained by means of Chernoff theorem. The estimates of the deviation of compositions of independent identically distributed random 1-parametric semigroup from its mean value is obtained. The convergence of the sequence of compositions of independent random 1-parametric semigroup to its mean value in probability is studied (see [2]). The approximation of the 1-parametric semigroup by the sequence of composition of random shifts operators is investigated. The property of asymptotically independence of Feynman iteration of the random semigroup is investigated. The independization of the random operator valued function is defined as the map of this random function into the sequence of random operator valued functions which has asymptotically independent increments. The examples of independization of random operator valued function are given.

References

[1] *Orlov Yu.N., Sakbaev V.Zh., Smolyanov O.G.* Unbounded random operators and Feynman formulae// Izvestiya: Mathematics. 2016. **80**:6. P. 1131-1158.

[2] *Sakbaev V.Zh.* On the law of large numbers for compositions of independent random semigroups // Russian Mathematics **60**(10):72-76. October 2016.

Integrable Dissipative Dynamical Systems

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We study nonconservative systems for which the usual methods of the study, e.g., Hamiltonian systems, are inapplicable. Thus, for such systems, we must “directly” integrate the main equation of dynamics. We generalize previously known cases and obtain new cases of the complete integrability in transcendental functions of the equation of dynamics of a multi-dimensional rigid body in a nonconservative force field (see also [1]).

We obtain a series of complete integrable nonconservative dynamical systems with nontrivial symmetries. Moreover, in almost all cases, all first integrals are expressed through finite combinations of elementary functions; these first integrals are transcendental functions of their variables. In this case, the transcendence is understood in the sense of complex analysis, when the analytic continuation of a function into the complex plane has essentially singular points. This fact is caused by the existence of attracting and repelling limit sets in the system (for example, attracting and repelling focuses) (see also [2]).

Problems examined are described by dynamical systems with so-called variable dissipation with zero mean. The problem of the search for complete sets of transcendental first integrals of systems with dissipation is quite topical; a large number of works are devoted to it. Due to the existence of nontrivial symmetry groups of such systems, we can prove that these systems possess variable dissipation with zero mean, which means that on the average for a period with respect to the periodic coordinate, the dissipation in the system is equal to zero, although in various domains of the phase space, either the energy pumping or dissipation can occur. As applications, we study dynamical equations of motion arising in the study of the plane and spatial dynamics of a rigid body interacting with a medium and also a possible generalization of the obtained methods for the study of general systems arising in the qualitative theory of ordinary differential equations, in the theory of dynamical systems, and also in oscillation theory.

This activity is also devoted to general aspects of the integrability of dynamical systems with variable dissipation. First, we propose a descriptive characteristic of such systems. The term "variable dissipation" refers to the possibility of alternation of its sign rather than to the value of the dissipation coefficient (therefore, it is more reasonable to use the term "sign-alternating") (see also [3]).

The assertions obtained in the work for variable dissipation system are a continuation of the Poincaré–Bendixon theory for systems on closed two-dimensional manifolds and the topological classification of such systems.

The problems considered in the work stimulate the development of qualitative tools of studying, and, therefore, in a natural way, there arises a qualitative variable dissipation system theory.

Following Poincaré, we improve some qualitative methods for finding key trajectories, i.e., the trajectories such that the global qualitative location of all other trajectories depends on the location and the topological type of these trajectories. Therefore, we can naturally pass to a complete qualitative study of the dynamical system considered in the whole phase space. We also obtain condition for existence of the bifurcation birth stable and unstable limit cycles for the systems describing the body motion in a resisting medium under the streamline flow around. We find methods for finding any closed trajectories in the phase spaces of such systems and also present criteria for the absence of any such trajectories. We extend the Poincaré topographical plane system theory and the comparison system theory to the spatial case. We study some elements of the theory of monotone vector fields on orientable surfaces.

References

[1] M. V. Shamolin, A New Case of Integrability in the Dynamics of a Multidimensional Solid in a Nonconservative Field under the Assumption of Linear Damping, *Doklady Physics*, **59:8** (2014), 375–378

[2] M. V. Shamolin, A Multidimensional Pendulum in a Nonconservative Force Field, *Doklady Physics*, **60:1** (2015), 34–38

[3] M. V. Shamolin, New Cases of Integrable Systems with Dissipation on a Tangent Bundle of a Multidimensional Sphere, *Doklady Physics*, **62:5** (2017), 262–265

Shape and Strong Shape of Limit Sets in Dynamical Systems

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For the analysis of the long-term behaviour of a dynamical system. it is important to know how the attractor looks like. Since, attractors could have very complicated local structure, an appropriate tool is shape $\equiv$ a homotopy theory adapted to this kind of spaces. One of the most significant results is the following theorem

Theorem. *Suppose $\Phi : X \times R^+ \to X$ be a semiflow defined on a compact metric space X. If M is a global attractor then the inclusion $M \to X$ induces a shape equivalence.*

In the talk wil be given a short introduction to intrinsic approach to shape and usimg this approach the theorem will be proven for arbitrary metric spaces. By intrinsic approach (intrinsic shape) will be shown that a strong version of the theorem above can be proven i.e. shape can be replaced by stong shape. Similiar results, for other limit sets in dynamical systems like chain reccurent set are stated also.

Sparkling Saddle Loops of Vector Fields on Surfaces

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A vector field v on a compact two-dimensional surface different from the sphere and projective plane can have a dissipative saddle P with a saddle loop that attracts the "free" unstable separatrix of this saddle. When the loop is unfolded in a generic one-parameter family, new saddle loops appear for parameter values arbitrarily close to zero.

We study this as a semi-local bifurcation, i.e., we look at a small neighborhood of the unstable manifold of the saddle that has a loop. In the orientable case, when this neighborhood is a torus with a disk removed, we get the following result.

Theorem. *Let v be a smooth vector field as described above. Then for a generic one-parameter smooth local family $V = \{v_\varepsilon\}_{\varepsilon\in(\mathbb{R},0)}$ with $v_0 = v$ there is a small neighborhood U of $W^u(P)$ such that for the restriction $\overline{V}$ of V on U the bifurcation diagram is a Cantor set K that contains zero and lies either to the right or to the left from it. The endpoints of the intervals of $(\mathbb{R}, 0) \setminus K$ correspond to vector fields with separatrix loops. Moreover, family $\overline{V}$ is topologically equivalent to any other local family obtained in the same way; in particular, it is structurally stable.*

By modifying the field far from the unstable manifold of our saddle, we can obtain countably many nonequivalent bifurcation diagrams for local one-parameter families.

Theorem. *Let M be a compact smooth two-dimensional surface other than the sphere, Klein bottle, and projective plane. Then generic one-parameter local families of smooth vector fields on M admit at least countably many non-equivalent bifurcation diagrams.*

In our construction, each of these diagrams is obtained as the union of a cantor set K, of the same origin as above, and a countable set of points that intersects each inner interval of $(\mathbb{R}, 0) \setminus K$ by the same finite number of points. This latter number distinguishes the diagrams. For each of these diagrams, there is an open set of local families that realize it.

Chaos and Bifurcations in a System for Predators Competing for One Prey

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We give a review of some bifurcations and chaotic behaviour of n competing predators feeding on the same prey in a system of the type

$$X_i' = p_i\,\varphi_i(S)\,X_i - d_i\,X_i, \quad i = 1, ..., n, \tag{1a}$$

$$S' = H(S) - \sum_{i=1}^{n} q_i\,\varphi_i(S)\,X_i, \tag{1b}$$

where the variable S represents the prey and the variables X_i represent the predators. They are, of course, non-negative. The function φ_i is assumed non-decreasing. We assume

$H(0) = H(K) = 0$ for some $K > 0$, $H'(K) < 0$, $H''(s) < 0$, $\varphi_i(0) = 0$, $\varphi_i'(s) > 0$.

The functions φ_i and H are of the class $C^2[0, \infty)$.

We mainly consider the case where

$$H(S) = r\,S\left(1 - \frac{S}{K}\right),\ \varphi_i(S) = \frac{S}{S + A_i}, \tag{2}$$

and where the parameters r, K and A_i are positive.

In this case we assume $p_i > d_i$. If not, the corresponding predator will not survive.

The dynamics in the coordinate planes representing one of the predators and the prey is well known. But there are still many open questions when predators coexist. The system has no equilibrium, where predators coexist. But the predators can coexist in a cyclic or chaotic way. Different types of complicated chaos can occur even for biologically realistic parameters. We review some results on simple local behaviour, conditions for extinction of some of predators. We discuss different types of chaos, among them the normal period doubling and "spiral" chaos. We give some results on building discrete models approximating a Poincaré map.

References

[1] sipov, A, V., Söderbacka, G. *Poincaré map construction for some classic two predators - one prey systems.* Internat. J. Bifur. Chaos Appl. Sci. Engrg 27, B№1, 1750116, 9 pp, (2017).

[2] sipov, A, V., Söderbacka, G. *Review of results on a system of type many predators- one prey*, Nonlinear Systems and Their Remarkable Mathematical Structures, pp 520-540, CRC Press, to appear 2018.

Bifurcation Analysis of the Dynamics of Two Vortices

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We are concerning with a system two point vortices in a Bose - Einstein condensate enclosed in a trap [1]. The Hamiltonian form of equations of motion is presented and its Liouville integrability is shown. A bifurcation diagram is constructed, analysis of bifurcations of Liouville tori is carried out for the both cases of opposite-signed vortices, and same-signed vortices [2-3]. The types of critical motions are identified. Two-parameter family of integrable Hamiltonians is presented.

References

[1] Torres, P. J., Kevrekidis, P.G., Frantzeskakis, D. J., Carretero-GonzBŕalez, R., Schmelcher, P., and Hall, D. S., *Dynamics of Vortex Dipoles in Confined Bose - Einstein Condensates*, Phys. Lett. A, 2011, vol. 375, no. 33, pp. 3044-3050.

[2] Sokolov, S. V., and Ryabov P. E., *Bifurcation Analysis of the Dynamics of Two Vortices in a Bose–Einstein Condensate. The Case of Intensities of Opposite Signs*, Regular and Chaotic Dynamics, 2017, Vol. 22, No. 8, pp. 976-995.

[3] Sokolov, S. V., and Ryabov P. E., *Bifurcation Diagram of the Two Vortices in a Bose-Einstein Condensate with Intensities of the Same Signs*, Doklady Mathematics, 2018, Vol. 97, Issue 3, pp. 286-290.

On a Quantum Heavy Particle

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We consider Schrödinger equation for a particle on a flat n-torus in a bounded potential, depending on time. Mass of the particle equals $1/\mu^2$, where μ is a small parameter. We show that the Sobolev H^ν-norms, $\nu \geq 1$ of the wave function grow approximately as t^ν on the time interval $t \in [0, t_*]$, where t_* is slightly less than $O(1/\mu)$.

On Partially Hyperbolic Symplectic Automorphisms of a 4-dimensional Torus generated by integer unitary symplectic matricies

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We study partially hyperbolic symplectic automorphisms of a 4-dimensional torus generated by integer unitary symplectic matricies. Before only hyperbolic torus maps were mainly studied, the investigations of partially hyperbolic automorphisms started rather recently and there are still many open questions. In this paper, we give the classification of the linear symplectic automorphisms of a 4-dimensional symplectic torus considering both possible cases: transitive and intransitive. Also we construct examples of partially hyperbolic symplectic maps on the four-dimensional torus with various dynamics. In particular, we construct such map on the torus which has transitive one-dimensional foliations generated by the eigendirections corresponding to eigenvalues lying on the unit circle. In order to construct such an example, the theory of irreducible polynomials with integer coefficients and some properties of invariant subspaces of the related integer unitary matrices are exploited.

Levy Laplacian and Yang-Mills Heat Flow

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The Levy Laplacian Δ_L, as it was introduced by Paul Levy (see [1]), acts on the function f defined on the Hilbert space by the formula

$$\Delta_L f(x) = \lim_{n\to\infty} \frac{1}{n} < f''(x)e_k, e_k >,$$

where $\{e_n\}$ is an orthonormal basis in the Hilbert space. Also the original Levy Laplacian can been defined as the integral functional determined by the special form of the second derivative. In the modern literature the infinite dimensional Laplacians defined by the analogy of both of these definitions on the various functional spaces are also called the Levy Laplacians. The relationship between the different definitions of the Levy Laplacian on the same functional space can be non-trivial.

One of the main reasons of the interest in the Levy Laplacian is its relationship with the Yang-Mills equations. In [2] it was proved that the connection A with the curvature F satisfies the Yang-Mills equations

$$\nabla_\nu F^{\nu}{}_{\mu} = 0$$

if and only if the parallel transport U^A associated with the connection A is a harmonic functional for the Levy Laplacian:

$$\Delta_L U^A = 0.$$

In [2] the connection A on the flat space and the Levy Laplacian defined as the integral functional were considered. Some results on the relationship with the Yang-Mills equations and the Yang-Mills equations were also obtained in [3,4,5].

Our report is devoted to the Levy Laplacian on the infinite dimensional manifold. We show the equivalence of the different definitions of this operator. Previously the heat semi-group for the Levy Laplacian on the manifold was studied in [6]. We study the relationship between the heat equation for the Levy Laplacian on the manifold and the Yang-Mills heat flow. Namely, we show that the time depended connection $A(\cdot,t)$ satisfies the Yang-Mills heat equations

$$\frac{\partial}{\partial t} A_\mu(\cdot,t) = \nabla_\nu F^{\nu}{}_{\mu}(\cdot,t)$$

if and only if the associated flow of parallel transports $U(t) = U^{A(t)}$ satisfies the heat equation for the Levy Laplacian:

$$\frac{\partial}{\partial t} U(t) = \Delta_L U(t).$$

References

[1] *Lévy P.* Problèmes concrets d'analyse fonctionnelle. Paris, Gautier-Villars, 1951.

[2] *Accardi L., Gibilisco P., Volovich I. V.* Yang-Mills gauge fields as harmonic functions for the Lévy-Laplacian // Russ. J. Math. Phys. 1994. Vol. 2., No. 2, pp. 235–250.

[3] *Leandre R., Volovich I. V.* The Stochastic Lévy Laplacian and Yang-Mills equation on manifolds // Infin. Dimens. Anal. Quantum Probab. Relat. Top. 2001. Vol. 2., No. 2, pp. 151–172.

[4] *Volkov B. O.* Lévy Laplacians and instantons // Proceedings of the Steklov Institute of Mathematics. 2015. Vol. 290, pp. 210-222.

[5] *Volkov B. O.* Stochastic Levy differential operators and Yang-Mills equations // Infin. Dimens. Anal. Quantum Probab. Relat. Top. 2017. Vol. 20, No. 2, 1750008

[6] *Accardi L., Smolyanov O. G.* Feynman formulas for evolution equations with Levy Laplacians on infinite-dimensional manifolds, Doklady Mathematics 2006. Vol. 73 pp. 252–257.

Torus Bifurcation in System with 2:1 Resonance

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In the first papers of this series [10, 11] we studied a dissipative Hopf – Hopf bifurcation with 2 :1 resonance . In this paper, we continue the study of this bifurcation. We consider the four – dimensional autonomous system of differential equations given by

$$\dot{x} = F(x, \mu), \tag{1}$$

which depends on the real parameter μ. We assume that is $F(0, \mu) = 0$. We suppose that for a certain value of μ, say $\mu = 0$, the matrix $DF_x(0, 0)$ has two pairs of simple pure imaginary eigenvalues$\pm i\omega_1$and $\pm i\omega_2$. In this paper we consider 2:1 resonance $\omega_2/\omega_1 = 2$. This problem has been studied by many authors [2, 4, 6, 7, 9, 10]. We consider the secondary bifurcation (Neimark - Sacker bifurcation) of mixed - mode solution and dynamics of full system. A parameter dependent polynomial truncated normal form is derived. We study this truncated normal form :

$$\left\{ \begin{array}{lcl} \dot{X} & = & \varepsilon(X + \delta Y + \cos(\theta_0)\, Z + 2\, Y^2), \\ \dot{Y} & = & \varepsilon(-\delta X + Y + \sin(\theta_0)\, Z - 2\, X\, Y), \\ \dot{Z} & = & \varepsilon(2\,(-\gamma + X)\, Z). \end{array} \right. \tag{2}$$

The three-dimensional system (2) is the main object of study in this paper. It has been shown that system (2) demonstrate Hopf bifurcation, period–doubling cascades and chaotic attractor [1, 3, 5, 7, 8]. But a analytical study of Hopf bifurcation in this system wasn't given in this papers. In [21] we consider system (2) in the case $\theta_0 = \pi$. In this paper the system (2) is considered with conditions $\theta_0 \neq \pi$.

References

[1] N. Bussac. The Nonlinear three-wave system. Strange attractors and asymptotic solutions, Physica Scripta, 1982,v. T2/1,110-118.

[2] E. Dupuis, De L'Existence D'hypertores Pres D'Une Bifurcation de Hopf - Hopf avec resonance 1:2 Universitate d'Ottawa, 2000
[3] D.W. Hughes, R.E. Proctor Chaos and the effect of noise in a model of three-wave mode coupling, Physica D 1990, 46, 163 -176
[4] E. Knobloch, R.E. Proctor The Double Hopf bifurcation with
2 : 1 resonance. Proc.R.Soc.Lond. A,1988, 415, 61-90
[5] Y.A Kuznetsov Elements of applied bifurcation theory. Vol 112. Springer-Verlag, New York, 3 edition, 2004
[6] V.G. LeBlanc, W.F. Langford Classification and unfoldings of 1:2 resonant Hopf bifurcation. Arch.Rational Mech. Anal.1996,136,305-357
[7] V.G.LeBlanc, On some secondary bifurcations near resonant Hopf-Hopf interactions, Contin.Discrete Impuls.Systems 2000, 7, 405-427
[8] O.Lopez - Rebollal, J.R.Sanmartin A generic, hard transition to chaos Physica D 1995, 89, 204-221
[9] G. Revel, D.M. Alonso, J.L. Moiola, Numerical semi-global analysis of a 1:2 resonant Hopf-Hopf bifurcation,Physica D.,2013,247,40-53
[10] D. Yu. Volkov, The Andronov-Hopf Bifurcation with 2: 1 Resonance. Journal of Mathematical Sciences,2005, 128(2), 2831-2834.
[11] D. Yu. Volkov. The Hopf – Hopf bifurcation with 2:1 resonance: Periodic solutions and invariant tori. Conference Publications, 2015, 2015(special) : 1098-1104. doi: 10.3934/proc.2015.1098

Vector Bundles and Riemann–Hilbert–Birkhoff Problem

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Applications of the theory of holomorphic vector bundles with meromorphic connections to the classical Riemann–Hilbert problem are well known. We are going to apply holomorphic vector bundles with meromorphic additive shift or q-shift to studying of generalized Riemann–Hilbert–Birkhoff problem for difference and q-difference systems.

As the application of this approach we obtain a generalization of Birkhoff's existence theorem. We prove that for any admissible set of characteristic constants and monodromy there exists a system

$$Y(z+1) = A(z)Y(z) \quad \text{or} \quad Y(qz) = Q(z)Y(z), \tag{1}$$

which has the given monodromy and characteristic constants.

An Unguided Tour Started from Chirality

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We will report an unguided mathematical tour started from research on chirality since 2000 and, attracted by questions around attractors, led to a zigzag path across topology and dynamics, often switched dimensions.

People who joined this tour at various stages include F. Ding, B. Jiang, Y. Liu, Y. Ni, J. Pan, H. Sun, C. Wang, S. Wang, J. Yao, Y. Zhang, H. Zheng, Q. Zhou and B. Zimmermann. Conversations with R. Edwards, L. Wen, C. Bonatti, J. Hillman, S. Kamada and others, added to the twists and turns that made the trip more fun.

References

[1] B.J. Jiang, S.C. Wang, *Achirality and planarity,* Commun. Contemp. Math. 2 (2000), no. 3, 299–305.

[2] B.J. Jiang, Y. Ni, S.C. Wang, *3-manifolds that admit knotted solenoids as attractors.* Trans. Amer. Math. Soc. 356 (2004), no. 11, 4371–4382

[3] B.J. Jiang, Y. Ni, S.C. Wang, Q. Zhou, *Embedding infinite cyclic covers of knot spaces into 3-space.* Topology 45 (2006), no. 4, 691–705.

[4] B.J. Jiang, S.C. Wang, H. Zheng, *No embeddings of solenoids into surfaces.* Proc. Amer. Math. Soc. 136 (2008), no. 10, 3697–3700.

[5] F. Ding, J.Z. Pan, S.C. Wang, J.G. Yao, *Manifolds with $\Omega(f)$ a union of DE attractors are rational homology spheres.* Ergodic Theory Dynam. Systems 30 (2010), no. 5, 1399-1417.

[6] B.J. Jiang, S.C. Wang, Q. Zhou, H. Zheng, *On tame embeddings of solenoids into 3-space.* Fund. Math. 214 (2011), no. 1, 57-75

[7] F. Ding, Y. Liu, S.C. Wang, J.G. Yao, *Extending T^p automorphisms over R^{p+2} and realizing DE attractors.* Discrete and Continuous Dynamical Systems, (2012) Volume 32, 1639 - 1655.

[8] F. Ding, Y. Liu, S. Wang, J. Yao, *Spin obstruction for codimension-two diffeomorphism extension.* Math. Res. Lett. 19 (2012), no. 2, 345-357.

[9] C. Wang, S. C. Wang, Y. M. Zhang, B. Zimmermann, *Embedding surfaces into S^3 with maximum symmetries.* Groups Geom. Dyn. 9 (2015), no. 4, 1001-1045.

On the Concept of Lyapunov's Transformations Groups

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The purpose of studying the groups of Lyapunov transformations is to classify differential equations according to certain asymptotic properties of solutions [1–3]. Using the concept of asymptotic equivalence (according to Lyapunov), a partition of a certain class of differential equations into equivalence classes is carried out. In turn, the purpose of such a classification is to obtain, if possible, a complete set of invariants of the Lyapunov transformation group. When using the group method to search for exact solutions of ordinary differential equations, if possible, or to study the properties of the solutions of the latter, a certain transformation is sought, leading the studied equation to a standard form for which the integration methods or the required properties of the solutions are known. More precisely: a certain group of transformations acts on the entire class of differential equations of a given type; an element of this group transforms the original equation into an equation of the same class and, accordingly, the solution of the equation under study into a solution of the transformed equation, i.e. the set of elements of the group of transformations is closed on a given class of equations and their solutions with respect to the action of the composition of the elements of this group of transformations. Developing this approach, one can write out all canonical forms of solvable (for example, in quadrature) equations and, applying all sorts of (reversible) transformations to them, find an infinite number of other solvable equations. The same can be applied to equations for which the properties of their solutions are simply known. It is quite natural that in this case there arise questions of describing the fullest possible system of invariants of the transformations under consideration. Definition. A Lyapunov transformation group is a group of transformations of a certain set of differential equations, the list of invariants of which contains the stability of the trivial solution and the spectrum. In contrast to the linear case, in the situation mentioned above it is not always possible to guarantee the uniformity of the exponential estimates of solutions. Consequently, the conclusions on stability, for example, the zero solution, which follow from these estimates will not always be true. If, however, for evaluating solutions, there is a uniformity over the initial point, then in this case the conclusions are the same as for linear systems. The group approach described above, of course, is not without some flaws, and the concept of the Lyapunov transformation also does not always allow solving the problem of the behavior of solutions, for example, when the spectrum of the equation contains zero. In this case, the simplest equation of a particular equivalence class will be associated with the problem of characteristic exponents, and some unified approach will be needed to the study of properties of solutions, which may be different from the use of transformation groups. Also, when using the group approach, there is no guarantee that the particular equation of interest to us will be in the set in a certain sense of equivalent differential equations. Then the task of finding the necessary, as well as sufficient conditions, or even the criteria for the reducibility of given classes of differential equations to equations of a particular type, is brought to the fore. As for the areas of application of the Lyapunov transformation groups, the following can be noted. Namely: the mathematical description of complex diffuse systems in the natural sciences (for the purpose of studying them) cannot be produced by deterministic laws. In such situations it is necessary to use suitable mathematical models. The use of classical methods of research and integration of ordinary differential equations in various areas of mathematical modeling is not effective enough to solve a large number of applied problems. For example, classical methods are ineffective when used in the study of many natural sciences and differential models in education, in the application of differential equations in variational problems, in the numerical-analytical integration of equations that have a non-unique solution, in the study of certain inverse problems. And here the group approach comes to the fore. Most of the problems of describing technical and technological systems leads to too complex nonlinear differential equations that cannot be solved in an exact way. In such cases, it is possible to suggest approaches similar to those described in this work.

References

[1] Якупов З. Я. Ляпуновские преобразования стационарного уравнения Шрёдингера с возмущающим потенциалом// Математическое моделирование. 1995. Т.7(5). С. 71.

[2] Якупов З.Я. О преобразованиях Ляпунова// Теория функций, её приложения и смежные вопросы/ Труды Математического центра им. Н.И. Лобачевского. Т. 54. Казань: Изд-во Казан. Матем. общества, изд-во Академии наук РТ, 2017. С. 413-417.

[3] Якупов З. Я., Галимова Р.К. Основная концепция исследования групп преобразований Ляпунова// Вестник КГТУ им. А. Н. Туполева. 2017. № 4. С. 171-176.

The Period-set of a Map from the Cantor-Set to Itself

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[The talk is based on the joint work with James W. Cannon & Mark Meilstrup]
This piece of research was motivated by Sharkovskiĭ's Theorem, which shows that the periods of the periodic points of a self-map of the unit-interval are severely restricted in the sense, that only tails of a non-standard linear order of the natural numbers can be realized as period-sets. In this research project we asked the analogous question for the Cantor-Set: Which period-sets can be realized by a (continuous) self-map f of the Cantor-Set to itself? − Although the proof of Sharkovskiĭ's Theorem heavily relies on using the Intermediate Value Theorem which does by no means apply to the Cantor-Set, we came to the conclusion that some kind of restriction does even hold in case of the Cantor-Set: The talk will be devoted to state and sketch the proof of a Theorem confirming that, while an arbitrary subset of the natural numbers can occur as period-set of f as long as f is allowed to have aperiodic points or preperiodic points, a necessary and sufficient restriction for a set to become a period set of some f will be described for those f where each point of the Cantor set belongs to some period.

References

[1] J. W. Cannon, M. H. Meilstrup & A. Zastrow: “The period set of a map from the Cantor set to itself", Discrete Contin. Dyn. Syst., Vol. 33 (2013), no. 7, 2667–2679.

Some Questions of the Stability of the Stochastic Model of Population Growth

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One of the pressing issues of modeling systems that describe the dynamics of population numbers are issues of sustainability. For various deterministic models, the questions of stability and control have been studied in many papers since the last century. Studying the issues of population size management, even in the deterministic case, is a difficult and urgent task today [1]. The results of constructing the control and stability of stochastic evolutionary models are considered in [2, 3]. In this paper, we study a logistic stochastic differential equation that describes the dynamics of population growth in the form:

$$dx(t) = y(t)(x - \frac{x^2}{K})dt, \tag{1}$$

where $x(t)$ is the volume of the unstructured population.

The parameter K is called the population capacity, expressed in units of population (or concentration) and is determined by a number of different circumstances, among which there are restrictions on the amount of substrate for microorganisms, the available volume for a population of tissue cells, food base or shelters for higher animals.

It is assumed that $y(t)$ is a random process describing the natural growth of a population with an indefinite distribution density.

The main results of the work are the obtained conditions of asymptotic stability with respect to probability in general for the described model. The method of moments studied questions of exponential stability in the mean-square. Numerical illustrating examples were constructed in which the dependences of population size on random population growth factors were studied and areas of stability were constructed for different values of the distribution density. The conditions for the existence of an equilibrium of such a system are obtained depending on the distribution density of the random process $y(t)$.

References

[1] A. A. Davydov, A. S. Platov. Optimal exploitation of two competing populations structured by size. // Tr. Mekh. UMO RAS, 19, vol. 4, 2013, P. 89—94.

[2] L. I. Rodina. Optimization of the average temporal benefit for a probabilistic model of a population exposed to fishing. // Bulletin of Udmurt state University. Mathematics. Mechanics. Computer science., 2018, T. 28, vol. 1, P. 48—58.

[3] A. V. Demidova, O. V. Druzhinina, O. N. Masina. Investigation of the stability model of population dynamics based on the construction of stochastic self-consistent models and the principle of reduction. // Bulletin of RUDN. Series: Mathematics. Computer science. Physics. vol. 3, 2015. P. 18—29.

Topological Conjugacy of Pseudo-Anosov Homeomorphisms with Marked Points

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Let $f : M \to M$ be a pseudo-Anosov homeomorphism of a closed orientable surface. Let $S_1, S_2 \subset M$ be two finite f-invariant sets consisting of non-singular points of contracting and expanding foliations of f.

Question: Is there exists homeomorphism $h : M \to M$ such that $h(S_1) = S_2$ and $h \circ f = f \circ h$. This problem is of interest in connection with the problem of the topological conjugacy of diffeomorphisms of surfaces with one-dimensional hyperbolic attractors.

The report will describe how to adapt the algorithms given in the author's book "Topological conjugacy of pseudo-Anosov homeomorphisms" (in russian), M., MTSNMO, 2013, to solve this problem.

On Attractors of Foliations with Transverse Linear Connections

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The study of the dynamical properties of foliations is an actual area. The problems of the existence and structure of minimal sets of Cartan foliations are studied in [1]. The problem of the existence of attractors for foliations with transverse Cartan geometries are investigated in [2]. The work [3] is devoted to study the existence of attractors for Weyl foliations modelled on pseudo-Riemannian geometry.

In this work we investigate the problem of the existence of attractors for foliations with transverse linear connection.

We use the following most general notion of an attractor for a foliation.

Definition 1. *Let (M, F) be a foliation. A subset of a manifold M is called saturated if it is a union of leaves of this foliation. A nonempty closed saturated subset $\mathcal{M}$ of M is called an attractor of (M, F) if there exists an open saturated neighbourhood $\mathcal{U} = \mathcal{U}(\mathcal{M})$ of the set $\mathcal{M}$ such that the closure of every leaf from $\mathcal{U} \setminus \mathcal{M}$ contains the set $\mathcal{M}$. The neighbourhood $\mathcal{U}$ is uniquely determined by this condition and it is called the basin of this attractor; we denote it by $\mathcal{B}(\mathcal{M})$. If in addition $\mathcal{B}(\mathcal{M}) = M$, then the attractor $\mathcal{M}$ is called global.*

An attractor $\mathcal{M}$ of a foliation (M, F) is said to be *transitive* if there exists a leaf L which is dense in $\mathcal{M}$, i.e., if $\overline{L} = \mathcal{M}$.

Recall that *a minimal set* of a foliation on a manifold M is a nonempty saturated closed subset in M that has no proper subset satisfying this condition. Attractors coincided with minimal sets give examples of transitive attractors.

Recall that a subgroup of a Lie group G is called relatively compact, if its closure in G is compact. We denote by $K =< A >$ the group generated by A.

Since in general foliations with transverse linear connection don't admit an attractor, we specify conditions that guarantee the existence of attractors which are minimal sets of (M, F) and prove the following statement.

Theorem 1. *Let (M, F) be a foliation with transverse linear connection of codimension q on n-dimensional manifold M, $0 < q < n$. Suppose that there exists a leaf L such that its linear holonomy group contains an element defined by a matrix of the form $D \cdot A$, where $K =< A >$ is a relatively compact subgroup in the linear group $GL(q, \mathbb{R})$ and $D = \operatorname{diag}(d_1, ..., d_q)$ with $|d_i| < 1$ for $1 \leq i \leq q$. Then:*

(i) the foliation (M, F) has an attractor $\mathcal{M} = \overline{L}$ which is a minimal set;

(ii) if moreover, (M, F) admits an Ehresmann connection, then $\mathcal{M} = \overline{L}$ is a global attractor and a minimal set of (M, F).

A leaf L is referred to as proper, if L is an embedded submanifold of M.

Corollary 1. *Let (M, F) be a foliation with transverse linear connection having a proper leaf L satisfying the conditions of Theorem 1. Then:*

(i) the leaf L is closed and an attractor of (M, F);

(ii) if moreover, (M, F) admits an Ehresmann connection, then L is the unique closed leaf and a global attractor of (M, F).

An application to transversally similar pseudo-Riemannian foliations is considered. We also describe the global structure of transversally similar Riemannian foliations which are not Riemannian and investigate the structure of their global attractors.

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References

[1] Zhukova, N. I. Minimal sets of Cartan foliations. Proc. Steklov Inst. Math. 2007. V. 256, no. 1. P. 105–135

[2] Galaev A. S., Zhukova N. I. Attractors of Cartan foliations // ArXiv:1703.07597 [math.DG]. 2017. P. 1– 21.

[3] Zhukova N. The existence of attractors of Weyl foliations modelled on pseudo-Riemannian manifolds // Journal of Physics: Conference Series. 2018. V. 990, no. 1. P. 1–15

Finitely additive measures on the invariant foliations of Anosov diffeomorphisms

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For a C^3 smooth topologically mixing Anosov diffeomorphism $F : M \to M$ with oriented invariant foliations, we prove the effective equidistribution theorem for the leafwise averages of C^2 functions on the (iterated) unstable leaves. The key tool is the analysis of the spectrum of the transfer operator acting on an appropriate Banach space $\mathfrak{B}$ of currents of degree k, where k is the dimension of the unstable foliation.

The elements of $\mathfrak{B}$ give rise to special families of distributions on the unstable leaves called *finitely additive measures.* We give a (partial) classification of the finitely additive measures, invariant under the stable holonomy map, in terms of the spectrum of the transfer operator.

We show that the asymptotics of the leafwise averages of C^2 smooth functions on the iterated unstable balls are controlled by the holonomy invariant finitely additive measures.

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Геометрия слоений полиномиального роста с конечной топологией слоев на 3-многообразиях

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В работе рассматриваются замкнутые ориентируемые 3-мерные многообразия с гладким ориентируемым слоением коразмерности один. Вводится новый класс слоений – это так называемые B-слоения, которые определяются следующим свойством слоев: слои B-слоения имеют ограниченную интегральную абсолютную кривизну $\int_L |K| d\mu < \infty$ в некоторой индуцированной метрике. Из результата Кон-Фоссена [1] следует, что B-слоения содержат слоения, допускающие неотрицательную кривизну слоев. Такие слоения были нами исследованы в [4]. Из результатов Хубера [2] следует, что B-слоения имеют конечную топологию слоев, т.е. слои гомеоморфны замкнутым поверхностям с конечным (возможно пустым) числом проколов. Кроме того, слои B-слоения должны иметь не более, чем квадратичный рост объема слоев [3]. Это позволяет нам доказать теорему, утверждающую, что B-слоения являются слоениями почти без голономии. Используя результаты Г. Эктора можно показать, что класс слоений почти без голономии совпадает с классом слоений имеющих полиномиальный рост слоев, если топология слоев конечна. Рассматривается обратная задача. Показывается, что не всякое слоение почти без голономии с конечной топологией слоев является B-слоением. В то же время доказывается, что обратное утверждение все же имеет место для собственных слоений. В частности, из представления 3-многообразия в виде открытой книги следует, что любое ориентируемое замкнутое 3-многообразие допускает B-слоение.

Список литературы

[1] Cohn-Vossen, Stefan."Küzeste Wege und Totalkrümmung auf Flächen." Compositio mathematica 2 (1935): 69-133.

[2] Huber, Alfred. "On subharmonic functions and differential geometry in the large." Commentarii Mathematici Helvetici 32.1 (1958): 13-72.

[3] Hartman, Philip. "Geodesic parallel coordinates in the large." American Journal of Mathematics 86.4 (1964): 705-727.

[4] Bolotov, Dmitry V. "Foliations of nonnegative curvature on closed 3-dimensional manifolds." Sbornik: Mathematics 200.3 (2009): 313.

Синтез оптимальных процессов со сменой уравнений движения

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Рассматриваестся задача оптимального управления с конечным числом переключений. В момент переключения происходит смена математической модели системы управления [1,2,3], а именно: меняются уравнения движения, пространство состояний, допустимые управления и т.п. Такие переключения, например, характерны для задач управления группами летательных аппаратов, когда изменяется количество управляемых объектов. Получены достаточные условия оптимальности, применение которых демонстрируется на академических примерах группового быстродействия.

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1. Постановка задачи. Пусть на заданном промежутке времени $T = [t_0, t_F]$ динамическая система совершает N переключений в моменты времени $t_1, \ldots, t_N$: $t_0 \leqslant t_1 \leqslant \ldots \leqslant t_N \leqslant t_F$. Между неравными последовательными моментами переключений состояние системы изменяется непрерывно:

$$\dot{x}_i(t) = f_i(t, x_i(t), u_i(t)), \quad t_i \leqslant t \leqslant t_{i+1}, \tag{1}$$

а в моменты переключений – дискретно:

$$x_i(t_i) = g_i(t_i, x_{i-1}(t_i), v_i), \quad i = 1, \ldots, N,$$

где $x_i(t) \in X_i \subset \mathbb{R}^{n_i}$ – состояние системы после i-го переключения, $u_i(t) \in U_i \subset \mathbb{R}^{p_i}$ – управление непрерывным движением, $v_i \in V_i \subset \mathbb{R}^{q_i}$ – управление переключением. На множестве допустимых процессов $\mathcal{D}_0(t_0, x_0)$ задан функционал

$$I_0^N(t_0, x_0, d) = \sum_{i=0}^{N} \int_{t_i}^{t_{i+1}} f_i^0(t, x_i(t), u_i(t))dt + \sum_{i=0}^{N} g_i^+(t_i, x_{i-1}(t_i), v_i) + F_N(x_N(t_F)). \tag{2}$$

Здесь (t_0, x_0) – начальное состояние, g_i^+ – неотрицательная функция. Требуется найти наименьшее значение функционала (1) и оптимальный процесс $d^* \in \mathcal{D}_0(t_0, x_0)$, на котором это значение достигается. Количество N и моменты переключений $t_1, \ldots, t_N$ заранее не заданы и находятся при минимизации функционала (2).

2. Метод решения. Обозначим через $\varphi_i(t, x_i)$ функцию цены (функцию Гамильтона – Якоби – Беллмана (ГЯБ)), равную минимальному значению функционала оставшихся потерь $I_i(t, x_i, d)$ на множестве $\mathcal{D}_i(t, x_i)$ допустимых процессов после i-го переключения. Определим *образующую* функции цены, значение $\varphi_i^k(t, x_i)$ которой равно минимальному значению функционала оставшихся потерь $I_i^k(t, x_i, d)$ на множестве $\mathcal{D}_i^k(t, x_i)$ допустимых процессов с k оставшимися переключениями после i-го. Наконец, *двухпозиционной* функцией цены $\phi_i(\theta, x_{i\,\theta}|\tau, x_{i\,\tau})$ будем называть решение задачи Лагранжа для системы (1) с фиксированными концами траектории

$$x_i(\theta) = x_{i\,\theta}, \quad x_i(\tau) = x_{i\,\tau}, \quad \int_\theta^\tau f_i^0(t, x_i(t), u_i(t))dt \to min\,.$$

Эта функция удовлетворяет уравнению ГЯБ с нулевым терминальным условием

$$\min_{u_i \in U_i} \left[\frac{\partial \phi_i}{\partial t} + \frac{\partial \phi_i}{\partial x_i} f_i(t, x_i, u_i) + f_i^0(t, x_i, u_j) \right] = 0, \quad \phi_i(\tau, x_i)|\tau, x_i) = 0. \tag{3}$$

Вспомогательные функции связаны между собой и "настоящей" функцией цены равенствами

$$\varphi_i(t, x_i) = \min_{k \in \mathbb{Z}_+} \varphi_i^k(t, x_i), \tag{4}$$

$$\varphi_i^k(t, x_i) = \min_{t \leqslant \tau \leqslant t_F} \min_{y \in X_i} \left\{ \phi_i^k(t, x_i|\tau, y) + \min_{v \in V_{i+1}} \left[\varphi_{i+1}^{k-1}(\tau, g_{i+1}(\tau, y, v) + g_{i+1}^+(\tau, y, v) \right] \right\}. \tag{5}$$

Процедура решения уравнений (3)–(5) начинается с нулевых образующих $\varphi_i^0(t, x_i)$, которые удовлетворяет уравнению (3) с конечными условиями $\varphi_i^0(t_F, x_i) = F_i(x_i)$, $i \in \mathbb{Z}_+$. Остальные образующие находятся согласно рекуррентному уравнению (5), а функции цены – по формуле (4). Операции минимизации в (3)–(5), определяют оптимальные управления непрерывным движением и переключениями, количество и позиции переключений, что позволяет синтезировать оптимальные процессы.

Список литературы

[1] Величенко В.В. Оптимальное управление составными системами // Докл. АН СССР. 1967. Т. 176. № 4. С. 754-756.

[2] Болтянский В.Г. Задача оптимизации со сменой фазового пространства // Диф. уравнения. 1983. Т. 19. № 3. С. 518-521.

[3] Bortakovskii A.S. Synthesis of Optimal Control-Systems with a Change of the Models of Motion // J. Comput. Syst. Sci. Int. 2018. Vol. 57, No. 4, pp. 543–560.

О бэровской классификации топологической ε-энтропии неавтономных динамических систем

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Напомним определение топологической энтропии для неавтономных динамических систем [1]. Пусть (X, d)— компактное метрическое пространство, $F \equiv (f_1, f_2, \ldots)$— последовательность непрерывных отображений из X в X. Наряду с исходной метрикой d определим на X дополнительную систему метрик

$$d_n^F(x, y) = \max_{0 \le i \le n-1} d(f^{\circ i}(x), f^{\circ i}(y)),$$

$$(f^{\circ i} \equiv f_i \circ \cdots \circ f_1,\ f^{\circ 0} \equiv \mathrm{id}_X), \quad x, y \in X, \quad n \in \mathbf{N}.$$

Для всяких $n \in \mathbf{N}$ и $\varepsilon > 0$ обозначим через $N_d(F, \varepsilon, n)$ максимальное число точек в X, попарные d_n^F-расстояния между которыми больше, чем ε. Такой набор точек называется (F, ε, n)-отделенным. Тогда *топологической энтропией* неавтономной динамической системы $(X,\ F)$ называется величина

$$h_{\mathrm{top}}(F) = \lim_{\varepsilon \to 0} \limsup_{n \to \infty} \frac{1}{n} \ln N_d(F, \varepsilon, n). \tag{1}$$

Отметим, что топологическая энтропия не зависит от выбора метрики, порождающей на X данную топологию, поэтому определение (1) корректно.

По последовательности непрерывных по совокупности переменных отображений

$$F \equiv (f_1, f_2, \ldots), \quad f_i : \mathcal{M} \times X \to X \tag{2}$$

образуем функцию

$$\mu \mapsto h_{\mathrm{top}}(F(\mu, \cdot)). \tag{3}$$

При произвольных $\mathcal{M}$, X и для любой последовательности отображений (2) функция (3) принадлежит третьему бэровскому классу [2]. В случае, когда X — канторово совершенное множество и $\mathcal{M}$ — множество иррациональных чисел на отрезке $[0; 1]$ с метрикой, индуцированной стандартной метрикой вещественной прямой, найдется последовательность отображений (2), для которой функция (3) всюду разрывна и не принадлежит второму классу Бэра [2].

Наряду с функцией (3), для произвольного положительного ε рассмотрим функцию

$$\mu \mapsto h(F(\mu, \cdot), \varepsilon) \equiv \limsup_{n \to \infty} \frac{1}{n} \ln N_d(F, \varepsilon, n). \tag{4}$$

Теорема 1. *Для любой последовательности отображений (3) и для каждого $\varepsilon > 0$ функция (4) принадлежит второму бэровскому классу.*

Теорема 2. *Если $\mathcal{M}$ метризуемо полной метрикой, то для каждого $\varepsilon > 0$ множество точек полунепрерывности сверху функции (4) является всюду плотным множество типа G_δ.*

Список литературы

[1] Kolyada S., Snoha L'. Topological entropy of nonautonomous dynamical systems // Random&Computational dynamics. 1996. V. 4, № 2&3. P. 205–233.

[2] Ветохин А. Н. Бэровская классификация топологической энтропии неавтономных динамических систем // Дифференциальные уравнения. 2018. Т. 54, № 6. С. 847-847.

Применение крайних подаргументов и надаргументов к поиску глобальных экстремумов R-значных функций

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Пусть X — вещественное линейное пространство (ЛП) и D — его подмножество. Если задана функция $f : D \to \mathbf{R}$, то будем обозначать через $\mathrm{Gr}(f)$ ее *график* $\{(x, f(x)) \mid x \in D\}$, через $\mathrm{Sub}(f)$ — ее *подграфик* $\{(x,y) \in D \times \mathbf{R} \mid y \leq f(x)\}$, и через $\mathrm{Epi}(f)$ — ее *надграфик* $\{(x,y) \in D \times \mathbf{R} \mid y \geq f(x)\}$. *Глобальный максимум функции* f на D обозначим через $\max_D f$, а множество точек, где он достигается — через $\mathrm{Argmax}_D f$. Аналогично вводятся обозначения $\min_D f$ и $\mathrm{Argmin}_D f$. Через $\sup_D f$ и $\inf_D f$ обозначим, соответственно, *супремум* и *инфимум* функции f на D.

Напомним (см. [1, § 1.18]), что точка выпуклого множества называется *крайней*, если она не является внутренней ни для какого отрезка, лежащего в этом множестве. *Выпуклую оболочку* множества D будем обозначать через $\mathrm{Conv}(D)$ или $\widetilde{D}$, а *множество его крайних точек* — через $\mathrm{Extr}(D)$.

Введем понятия крайнего подаргумента и крайнего надаргумента.

Определение 1. *Пусть X — ЛП и $D \subset X$.* Крайним подаргументом *(соответственно* крайним надаргументом*) функции $f\colon D \to \mathbf{R}$ назовем аргумент x (то есть первую компоненту) любой крайней точки (x,y) множества* $\mathrm{Conv}(\mathrm{Sub}(f))$ *(соответственно* $\mathrm{Conv}(\mathrm{Epi}(f))$*). Множество всех крайних подаргументов (соответственно надаргументов) функции f на множестве D будем обозначать через* $\mathrm{ExtrSA}(f,D)$ *(соответственно* $\mathrm{ExtrEA}(f,D)$*).*

Замечание 1. *Очевидно, что выполняются равенства* $\min_D(f) = -\max_D(-f)$, $\mathrm{Argmin}_D(f) = \mathrm{Argmax}_D(-f)$ *и* $\mathrm{ExtrEA}(f,D) = \mathrm{ExtrSA}(-f,D)$. *Поэтому из любого утверждения про максимумы и крайние подаргументы вытекает двойственное утверждение про минимумы и крайние надаргументы (и наоборот).*

Предложение 1. *Если X — ЛП и $D \subset X$, то для любой функции $f\colon D \to \mathbf{R}$ верны включения* $\mathrm{Extr}(\widetilde{D}) \subset \mathrm{ExtrSA}(f,D) \subset D$ *и* $\mathrm{Extr}(\widetilde{D}) \subset \mathrm{ExtrEA}(f,D) \subset D$.

Теорема 1. *Пусть X — ЛП, $D \subset X$ и заданы функции $f\colon D \to \mathbf{R}$, $g\colon \widetilde{D} \to \mathbf{R}$, $v\colon \widetilde{D} \to \mathbf{R}$, причем v строго положительна на D. Пусть, кроме того, строго выпукла на $\widetilde{D}$ функция $(g-M)\cdot v$, где $M = \sup_D(f/v+g) < \infty$. Тогда все точки глобального максимума на D функции $f/v + g$ являются крайними подаргументами функции f на D, то есть $\mathrm{Argmax}_D(f/v+g) \subset \mathrm{ExtrSA}(f, D)$.*

Из теоремы 1, с учетом замечания 1, вытекает двойственная теорема.

Теорема 2. *Пусть X — ЛП, $D \subset X$ и заданы функции $f\colon D \to \mathbf{R}$, $g\colon \widetilde{D} \to \mathbf{R}$, $v\colon \widetilde{D} \to \mathbf{R}$, причем v строго положительна на D. Пусть, кроме того, строго вогнута на $\widetilde{D}$ функция $(g-m)\cdot v$, где $m = \inf_D(f/v+g) > -\infty$. Тогда все точки глобального минимума на D функции $f/v + g$ являются крайними надаргументами функции f на D, то есть $\mathrm{Argmin}_D(f/v+g) \subset \mathrm{ExtrEA}(f, D)$.*

Замечание 2. *Множества $\mathrm{ExtrSA}(f, D)$ и $\mathrm{ExtrEA}(f, D)$ не зависят от функций g и v. Поэтому, найдя эти множества один раз, можно применять их с помощью теорем 1 и 2 для поиска экстремумов функций $f/v + g$ при различных g и v.*

Метод поиска глобальных экстремумов. На основе полученных результатов можно построить следующий метод поиска глобальных экстремумов некоторых функций вида $f/v+g$, не требующий дифференцируемости f в какой-либо точке:

1) убеждаемся (например, с помощью теорем 1, 2 или аналогичных им утверждений), что точки глобального экстремума функции $f/v + g$ принадлежат множеству $\mathrm{ExtrSA}(f, D)$ или множеству $\mathrm{ExtrEA}(f, D)$;

2) находим нужное множество $\mathrm{ExtrSA}(f, D)$ или $\mathrm{ExtrEA}(f, D)$;

3) вычисляем глобальный экстремум функции $f/v + g$ на множестве $\mathrm{ExtrSA}(f, D)$ или $\mathrm{ExtrEA}(f, D)$.

Список литературы

[1] Е. С. Половинкин, М. В. Балашов *Элементы выпуклого и сильно выпуклого анализа* М. Физматлит. 2004. 416 с.

Самоорганизованная критичность в теории ближайшей окрестности элементов перколяционных систем

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В работе, представленной в докладе, в рамках теории ближайшей окрестности [1-3] элементов перколяционных кластеров аналитически и средствами компьютерного моделирования исследуются свойства кластерных систем с самоорганизацией.

При варьировании структуры матрицы, на которой создаётся перколяционная система, модификации условий соединения кластеров, изменении вида элементов и других параметров, как известно, меняется тип моделируемых задач [4-14]. Введение понятия ближайшей окрестности – перколяционных полей меньшего масштаба, «описанных» вокруг элементов исходной матрицы, – также приводит к новому типу перколяционных задач, кардинально расширяет возможности исследования структуры и свойств образующихся кластеров, позволяет детализировать изучение процессов их генезиса.

В докладе описана группа алгоритмов компьютерного моделирования перколяционных систем на случайных решётках мультимасштабных «узлов», состоящих из перколяционных кластеров меньшего масштаба произвольной степени вложения по образцу матрёшки. Помимо стандартного набора параметров перколяционных систем в работе методом Монте-Карло рассчитаны анизотропия, лакунарность и радиус гирации кластеров, индекс роста их мощности, а также информационная и первая корреляционная размерности спектра Реньи.

В работе аналитически определены меры (в смысле теории размерности) на множестве проводящих участков перколяционного поля; рассчитаны индексы, описывающие скейлинговое поведение энтропии их разбиения; введено представление об относительной степени упорядоченности структуры, показана пригодность этой величины для оценки дрейфа свойств ближайшей окрестности [1-3]. Расчёты проведены для двумерных решёток с трёхуровневыми вложениями в ближайших окрестностях элементов.

Отдельно исследуются свойства мультимасштабной системы перколяционных структур на введённых в [11] коврах Серпинского с гибридной разветвлённостью.

В развитие концепции ближайшей окрестности исследованы некоторые аспекты влияния процессов самоорганизации на свойства перколяционной системы. В докладе представлена компьютерная модель управления структурой перколяционных кластеров в процессе их формирования [15].

Перколяционные задачи с самоорганизацией – неотъемлемая составляющая теории самоорганизующейся критичности, предложенной в [16, 17], в первую очередь, для осмысления связи между локальной организацией структуры и механизмом развития критичности [3-10, 12, 13].

Как известно, к наиболее общим закономерностям эволюции перколяционных систем с взаимодействующими элементами относится существование в них неравновесных квазистационарных состояний, возникающим за счёт многомасштабных корреляций в пространстве и времени [18]: пространственные корреляции обнаруживают себя в структуре перколирующих фрактальных множеств вблизи порога протекания, временные – в движении к таким состояниям при медленных воздействиях на систему, позволяющих протекать процессам самоорганизации. При этом стремление к самоорганизующейся критичности приобретает универсальный характер, что можно понять в контексте принципа наименьшего действия [18].

Построение кластерной системы в модели проводится методом Монте-Карло с использованием итерационного алгоритма реализации взаимодействия её элементов для двух типов закона притяжения: с силами пропорциональными $\frac{1}{R}$ и $\frac{1}{R^2}$ [15]. В работе изучена зависимость структуры и свойств самоорганизующихся кластеров от скорости генерации системы, характерных значений длины корреляции, а также от степени самоорганизации. Для этого исследована их зависимость соответственно от количества частиц, генерируемых на перколяционном поле на каждом шаге создания бесконечного кластера, от максимального расстояния, на котором элементы системы могут объединяться в кластер, а также от количества актов взаимодействия частиц. Аппроксимацией результатов модельных экспериментов получены аналитические выражения для зависимости от этих параметров мощности бесконечного кластера, степени анизотропии, радиуса гирации и лакунарности [15].

Список литературы

[1] A. Herega. AIP Conf. Proc. 1683, 020071 (1-4) (2015).

[2] A. Herega, V. Sukhanov, V. Vyrovoy. AIP Conf. Proc. 1783, 020072 (1-4) (2016).

[3] A. Herega. The Selected Models of the Mesostructure of Composites: Percolation, Clusters, and Force Fields. Springer-Verlag, Heidelberg (2018).

[4] J. Feder. Fractals. Plenum Press, New York (1988).

[5] B. Mandelbrot. The Fractal Geometry of Nature. Freeman and Co., San Francisco (1982).

[6] I. M. Sokolov. Dimensions and other critical exponents in the percolation theory. Sov. Phys. Usp. 29, 924 (1986).

[7] B. I. Shklovskii, A. L. Efros. Electronic Properties of Doped Semiconductors. Springer-Verlag, Berlin, Heidelberg (1984).

[8] A. L. Efros. Physics and Geometry of Disorder. Percolation Theory. MIR Publishers, Moscow (1986).

[9] М. В. Меньшиков, С. А. Молчанов, А. Ф. Сидоренко. Теория перколяции и некоторые приложения. Итоги науки и техники ВИНИТИ. Сер. Теория вероятностей 24, 53 (1986).

[10] D. Stauffer, A. Aharony. Introduction to Percolation Theory. Taylor and Fransis, London (1994).

[11] A. Herega et al. Phys. Usp. 55 (5), 519 (2012).

[12] A. Herega. Nanomechanics Science and Technology 4 (2), 119 (2013).

[13] А. Герега. Физическая мезомеханика 16 (5), 87 (2013).

[14] A. Herega. Journal of Materials Science and Engineering A5 (11-12), 409 (2015).

[15] А. Герега, Ю. Крывченко. Доклады межд. конф. «Алгебраические и геометрические методы анализа» (Одесса) 1, 99 (2018).

[16] P. Bak, C. Tang, K. Wiesenfeld. Phys. Rev. Lett. 59, 381 (1987).

[17] P. Bak, C. Tang, K. Wiesenfeld. Phys. Rev. A 38, 364 (1988).

[18] L. Zelenyi, A. Milovanov. Phys. Usp. 47, 749 (2004).

Распознавание гомотопически нетривиальных траекторий для некоторых динамических систем

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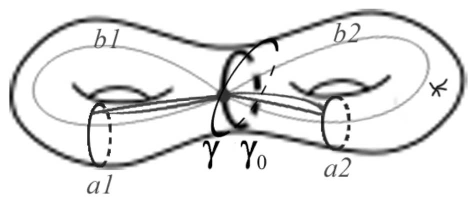
Рассмотрим замкнутую 2-мерную поверхность комплексной структуры, представленную в виде многоугольника с $2g$-сторонами, где g – это род поверхности. Рассмотрим бильярд в этом многоугольнике и траектории, начинающиеся на границе многоугольника. Среди них есть траектории, которые определяются конечным числом отражений, а на самой поверхности являются замкнутыми. В бильярде мы предполагаем, что угол падения равен углу отражения. Гомотопическую нетривиальность таких замкнутых траекторий можно детектировать с помощью итерированных интегралов от голоморфных и антиголоморфных форм на поверхности. Не все петли, представленные траекториями бильярда, будут гомотопически тривиальными. Например, для поверхности рода $g = 2$ существует траектория нашего бильярда, которая деформируется в горловину (гомотопна горловине). Эта траектория на самой поверхности представляет некоторую замкнутую кусочно-гладкую кривую. Её гомотопическая нетривиальность доказывается с помощью 2-итерированных интегралов. Крендель (поверхность рода 2), на котором рассматривается наша петля, в $\mathbb{C}^2$ задается следующим уравнением:

$$y^2 = (x - a_1) \cdot \ldots \cdot (x - a_5).$$



Возьмем голоморфную форму ω.

$$\omega = \frac{xdx}{\sqrt{(x - a_1) \cdot \ldots \cdot (x - a_5)}}.$$

Интеграл вдоль петель от этой голоморфной 1-формы равен нулю $\int_\gamma \omega = \int_{\gamma_0} \omega = 0$. Однако для 2-итерированного интеграла вдоль петель γ и γ_0, где $\gamma_0 = a_1 b_1 a_1^{-1} b_1^{-1} = a_2 b_2 a_2^{-1} b_2^{-1}$ имеем в силу свойств 2-итерированных интегралов

$$2 \int_\gamma \omega\overline{\omega} = 2 \int_{\gamma_0} \omega\overline{\omega} = \int_{a_1 b_1 a_1^{-1} b_1^{-1}} \omega\overline{\omega} + \int_{a_2 b_2 a_2^{-1} b_2^{-1}} \omega\overline{\omega} =$$

$$= \int_{a_1} \omega \int_{b_1} \overline{\omega} - \int_{a_1} \overline{\omega} \int_{b_1} \omega + \int_{a_2} \omega \int_{b_2} \overline{\omega} - \int_{a_2} \overline{\omega} \int_{b_2} \omega > 0.$$

Последнее неравенство следует из билинейных соотношений Римана. Итак, $\int_\gamma \omega\overline{\omega} \neq 0$, что влечет за собой негомотопность нулю петли γ, представленной траекторией бильярда. С помощью итерированных интегралов произвольной кратности мы можем детектировать гомотопическую нетривиальность петель, которые представляют собой траектории бильярда на поверхности.

Список литературы

[1] Драгович В., Раднович М. Интегрируемые бильярды, квадрики и многомерные поризмы Понселе. М.-Ижевск: Издательство «РХД», 2010.

[2] Спрингер Дж. Введение в теорию Римановых поверхностей. М.: Издательство иностранной литературы, 1960.

[3] Chen K. T. Iterated integrals of diffferential forms and loop space homology. Ann. Math. 97(1973), 217-246.

О бифуркации между различными типами соленоидальных базисных множеств

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В докладе рассматривается класс А-диффеоморфизмов Смейла-Виеториса, определяющийся с помощью базовых А-эндоморфизмов многообразий, размерность которых меньше размерности несущих многообразий А-диффеоморфизмов, и содержащий ДЕ-отображения Смейла. В общем случае неблуждающее множество А-диффеоморфизма Смейла-Виеториса не совпадает с инвариантным соленоидальным множеством, а разбивается на тривиальные и нетривиальные базисные множества. Здесь показано, что имеется взаимно однозначное соответствие между тривиальными (нетривиальными) базисными множествами базового А-эндоморфизма и А-диффеоморфизма Смейла-Виеториса. Для назад-инвариантного базисного множества базового А-эндоморфизма приводится точное описание соответствующего нетривиального базисного множества А-диффеоморфизма Смейла-Виеториса. На основе полученного описания стрятся бифуркации между различными типами соленоидальных базисных множеств. Эти бифуркации можно рассматривать как бифуркации разрушения (или рождения) соленоидальных базисных множеств. В докладе будет представлена бифуркация для 3-мерной сферы S^3.

Функции Морса на поверхностях с краем

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В данный работе мы рассматриваем простые функции Морса на двумерных поверхностях с краем. Пусть M – это двумерная компактная связная поверхность с заданной симплектической формой ω. Диффереоморфизм $\Phi : M \to M$ называется симплектоморфизмом, если он сохраняет форму ω. Основным результатом работы является классификация простых функций Морса относительно действия группы симплектоморфизмов. Оказывается, что полным инвариантом является граф Рибба функции, снабженной мерой некоторого специального вида.

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Список литературы

[1] Anton Izosimov, Boris Khesin, and Mehdi Mousavi, Coadjoint orbits of symplectic diffeomorphisms of surfaces and ideal hydrodynamics. Annales de l'Institut Fourier, 2016.

[2] Anton Isozimov and Boris Khesin, Classification of Casimirs in 2D Hydrodynamics. Moscow Mathematical Journal, 2017.

[3] Ilia Kirillov, Morse-Darboux lemma for surfaces with boundary. Journal of Geometry and Physics, 2018.

Математическая модель кризиса в педагогических системах

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Математические модели функционирования педагогических систем рассматриваются в работах Жегалова В.И., Киясова С.Н., Милованова В.П., Буданова В.Г., Чхартишвили А.Г. и др. В [1] предложена синергетическая модель педагогической системы в период кризиса. Дадим ее математическую интерпретацию.

Пусть $x(t)$ – планово-прогностическая деятельность системы, $y(t)$ – деятельность по созданию управленческой структуры, $z(t)$ – целеполагание. Состояние системы в момент кризиса опишем системой дифференциальных уравнений:

$$\begin{cases} \dot{x} = -kx + ay, \\ \dot{y} = bx + cy - dxz, \\ \dot{z} = -mz + lxy. \end{cases} \tag{1}$$

Скорость построения новой программы стабильного функционирования системы $\dot{x}(t)$ зависит от ее способности к планово-прогностической деятельности $x(t)$ и занятости созданием нового управления $y(t)$. Скорость формирования новой организационно-управленческой структуры $\dot{y}(t)$ определяется способностью системы к созданию структуры управления $y(t)$, занятостью планово-прогностической деятельностью $x(t)$ и ее взаимодействием с процессом целеполагания $z(t)$. Скорость построения "дерева целей" $\dot{z}(t)$ определяется способностью системы к целеполаганию $z(t)$, взаимодействием планово-прогностической деятельности $x(t)$ и деятельности по созданию новой управленческой структуры $y(t)$.

Коэффициенты k, a, b, c, d, m, l характеризуют внешнее и внутреннее влияние различных сред на систему. Они изменяются медленнее, чем создается новая целевая программа функционирования, поэтому считаем их постоянными. Эти коэффициенты являются управляющими (бифуркационными) параметрами и определяют саморегуляцию системы (1). Для их оценки проводились педагогические эксперименты в соответствии с методикой, описанной в [2].

Система (1) имеет три особые точки, определямые из условий $\dot{x}(t) = 0$, $\dot{y}(t) = 0$, $\dot{z}(t) = 0$. Одна – $O(0,0,0)$, координаты двух других выражаются через коэффициенты k, a, b, c, d, m, l. Характеристическое уравнение, соответствующее линеаризованной системе, при любых значениях параметров имеет два отрицательных и один положительный корень. Следовательно, точка $O(0,0,0)$ – седло-узел с двумерным устойчивым и одномерным неустойчивым инвариантными многообразиями.

Используя методы, изложенные в [3,4], с помощью численных экспериментов определены две гомоклинические траектории седло-узел, разрушение которых при изменении конкретных значений управляющих параметров приводит к бифуркации гомоклинического каскада (хаотического аттрактора).

При заданных начальных условиях $x(t_0) = x_0, y(t_0) = y_0, z(t_0) = z_0$ метод последовательных приближений [5] позволяет построить приближенное решение в виде функции, зависящей от x_0, y_0, z_0 и t с любой степенью точности. Результаты вычислений связаны с вопросом о существовании множества седловых предельных циклов и структуры аттрактора Лоренца.

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Список литературы

[1] Сибирев В. В., Сибирева А. Р. Структурная организация кризиса в педагогических системах // Казанский педагогический журнал. 2016. Т. 2, № 2. С. 1–7.

[2] Мандрикова Е. Ю. Разработка опросника самоорганизации деятельности (ОСД) // Психологическая диагностика. 2010. № 2. С. 59–83.

[3] Калошин Д. А. О построении бифуркационной поверхности существования гетероклинических контуров седло-фокусов в системе Лоренца // Дифференц. уравнения. 2004. Т. 40, № 12. С. 1705–1707.

[4] Гурина Т. А., Дорофеев И. А. Существование гомоклинической бабочки в модели устойчивости средней фирмы // Динамические системы. 2010. Вып. 28. С. 63–68.

[5] Афанасьев А. П., Дзюба С. М. Схема Горнера для исследования решений дифференциальных уравнений с полиномиальной правой частью // Бизнес Информатика. 2017. № 2(40). С. 33–39.

Бифуркационная диаграмма одной возмущенной задачи вихревой динамики

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В докладе рассматривается обобщение динамики системы двух вихревых нитей в бозе-эйнштейновском конденсате, заключенном в цилиндрической ловушке (см. [1]). Динамика системы двух точечных вихрей описывается системой обыкновенных дифференциальных уравнений относительно координат вихревых нитей, которая может быть представлена в гамильтоновой форме
$$\dot{\boldsymbol{\zeta}} = \{\boldsymbol{\zeta}, H\} \tag{1}$$
со стандартной скобкой Пуассона $\{x_i, y_j\} = -\frac{1}{\Gamma_i}\delta_{ij}$, где δ_{ij} – символ Кронекера, и функцией Гамильтона
$$\begin{aligned} H = {}& \ln[1-(x_1^2+y_1^2)] + a^2\ln[1-(x_2^2+y_2^2)] - ab\ln[(x_2-x_1)^2+(y_2-y_1)^2] \\ &+ a\varepsilon\ln[(x_1-x_2)^2+(y_1-y_2)^2+(x_1^2+y_1^2-1)(x_2^2+y_2^2-1)]. \end{aligned} \tag{2}$$

Здесь через (x_k, y_k) обозначены координаты k-ого вихря ($k = 1, 2$), фазовый вектор $\boldsymbol{\zeta}$ имеет координаты $\{x_1, y_1, x_2, y_2\}$, параметр a обозначает отношение интенсивностей. Физический параметр b характеризует меру взаимодействия вихрей и удерживающего потенциала. В работах [1], [2] на основе экспериментальных данных в случае равных интенсивностей принимались следующие значения параметра: $b = 2$; $b = 1,35$; $b = 0,1$. Интерес представляет изучение динамики для *произвольного* положительного значения параметра b.

Фазовое пространство $\mathcal{P}$ задается в виде прямого произведения двух открытых кругов единичного радиуса с выколотом множеством столкновениий вихрей. Система (1) допускает один дополнительный первый интеграл движения – *момент завихренности* $F = x_1^2+y_1^2+a(x_2^2+$

y_2^2). Функция F вместе с гамильтонианом H образуют на $\mathcal{P}$ полный инволютивный набор интегралов системы (1). Определим *отображение момента* $\mathcal{F} : \mathcal{P} \to \mathbb{R}^2$, полагая $(f, h) = \mathcal{F}(\boldsymbol{\zeta}) = (F(\boldsymbol{\zeta}), H(\boldsymbol{\zeta}))$. Обозначим через $\mathcal{C}$ совокупность всех критических точек отображения момента. Множество критических значений $\Sigma = \mathcal{F}(\mathcal{C} \cap \mathcal{P})$ называется *бифуркационной диаграммой.* В работах [3] и [4] для случая, когда $b = 1$ и $\varepsilon = 0$, исследована фазовая топология динамики двух вихревых нитей и их динамические эффекты.

Основная цель доклада – предъявить явную параметризацию бифуркационной диаграммы Σ для возмущенного гамильтониана (2), т.е когда параметр взаимодействия b принимает любые положительные значения. Обнаружены новые свойства бифуркационной диаграммы, которые ранее не встречались в задачах вихревой динамики [4], [5].

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Список литературы

[1] P. J. Torres, P. G. Kevrekidis, D. J. Frantzeskakis, R. Carretero-Gonzalez, P. Schmelcher, D. S. Hall. Dynamics of vortex dipoles in confined Bose – Einstein condensates // Phys. Lett. A. 2011. Vol. 375. P. 3044–3050.

[2] R. Navarro, R. Carretero-González, P. J. Torres, P. G. Kevrekidis, D. J. Frantzeskakis, M. W. Ray, E. Altuntaş, and D. S. Hall. Dynamics of Few Co-rotating Vortices in Bose-Einstein Condensates // Phys. Rev. Lett. 2013, May 31;110(22): 225301.

[3] S. V. Sokolov, P. E. Ryabov. Bifurcation Analysis of the Dynamics of Two Vortices in a Bose – Einstein Condensate. The Case of Intensities of Opposite Signs // Regular and Chaotic Dynamics, 2017, Vol. 22, No. 8, pp. 976–995.

[4] S. V. Sokolov, P. E. Ryabov. Bifurcation Diagram of the Two Vortices in a Bose-Einstein Condensate with Intensities of the Same Signs // Doklady Mathematics, 2018, Vol. 97, No. 3, pp. 1–5.

[5] Борисов А. В., Мамаев И. С. Математические методы динамики вихревых структур. Москва–Ижевск: Институт компьютерных исследований, 2005. 368 с.

О хаотических решениях разностных уравнений со случайными параметрами

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Разностные уравнения возникают при математическом моделировании дискретных динамических систем. Например, развитие многих биологических популяций с неперекрывающимися поколениями определяется уравнением

$$x_{n+1} = f(x_n), \quad n = 0, 1, \ldots, \tag{1}$$

где x_{n+1} — размер популяции в момент времени $n+1$ выражается через размер популяции x_n в предыдущий момент времени. Свойства решений таких уравнений описаны, в частности, в работах [1], [2].

Рассмотрим обобщение модели (1) в предположении, что в каждый момент времени n функция f зависит также от случайного параметра ω_n. Получим вероятностную модель, заданную разностным уравнением

$$x_{n+1} = f(\omega_n, x_n), \quad (\omega_n, x_n) \in \Omega \times I, \quad n = 0, 1, \dots, \tag{2}$$

где Ω — заданное множество с сигма-алгеброй подмножеств $\widetilde{\mathfrak{A}}$, на которой определена вероятностная мера $\widetilde{\mu}$, $I = [a, b]$.

Введем в рассмотрение вероятностное пространство $(\Sigma, \mathfrak{A}, \mu)$, где Σ означает множество последовательностей $\sigma = (\omega_0, \omega_1, \dots, \omega_n, \dots) \in \Omega^\infty$, система множеств $\mathfrak{A}$ является наименьшей сигма-алгеброй, порожденной цилиндрическими множествами $D_n \doteq \{\sigma \in \Sigma : \omega_0 \in \Omega_0, \dots, \omega_n \in \Omega_n\}$, где $\Omega_j \in \widetilde{\mathfrak{A}}$, $j = 0, \dots, n$, и определим меру $\widetilde{\mu}(D_n) = \widetilde{\mu}(\Omega_0) \cdot \widetilde{\mu}(\Omega_1) \cdot \ldots \cdot \widetilde{\mu}(\Omega_n)$. Тогда на измеримом пространстве $(\Sigma, \mathfrak{A})$ существует единственная вероятностная мера μ, которая является продолжением меры $\widetilde{\mu}$ на сигма-алгебру $\mathfrak{A}$. Для каждого $n \in \mathbb{N}$ обозначим

$$\sigma_n \doteq (\omega_0, \omega_1, \dots, \omega_{n-1}), \quad f^n(\sigma_n, x) \doteq f\big(\omega_{n-1}, \dots, f(\omega_1, f(\omega_0, x))\big).$$

Будем также пользоваться обозначениями $f^n(\sigma, x) \doteq f^n(\sigma_n, x)$ и $x_n(\sigma, x) \doteq f^n(\sigma, x)$.

Точки $\beta_0, \dots, \beta_{k-1}$ образуют *цикл B периода $k \geqslant 1$* для уравнения (2), если для всех $\sigma \in \Sigma$ выполнены равенства $f^k(\sigma, \beta_0) = \beta_0$, $f^m(\sigma, \beta_0) = \beta_m$, $m = 1, \dots, k-1$ и цикл B не содержит цикла меньшего периода.

Определение 1. *Цикл B назовем отталкивающим с вероятностью единица, если существуют множество $\Sigma_0 \subseteq \Sigma$ и окрестность U данного цикла, такие, что $\mu(\Sigma_0) = 1$ и для каждой точки $(\sigma, x) \in \Sigma_0 \times (U \setminus B)$ найдется номер $N = N(\sigma, x)$, для которого $f^N(\sigma, x) \notin U$.*

Определение 2. *Решение $x_n(\sigma, x_0)$ уравнения (2) (при фиксированном значении $\sigma \in \Sigma$) назовем хаотическим, если для каждого $k \in \mathbb{N}$ предел $\lim\limits_{n\to\infty} x_{nk}(\sigma, x_0)$ не существует. Точку $x_0 \in I$ назовем апериодической с вероятностью единица точкой уравнения (2), если существует множество $\Sigma_0 \subseteq \Sigma$ такое, что $\mu(\Sigma_0) = 1$ и для любого $\sigma \in \Sigma_0$ решения $x_n(\sigma, x_0)$ хаотические.*

Точку y назовем со временем периодической или предпериодической точкой уравнения (2), если существует $m \in \mathbb{N}$ такое, что для любых $\sigma_m \in \Omega^m$ точка $x = f^m(\sigma_m, y)$ является точкой некоторого периода $k \geqslant 1$.

Теорема. *Предположим, что уравнение (2) либо не имеет ни одного цикла (периода $k \geqslant 1$), либо все циклы отталкивающие с вероятностью единица. Пусть Y — множество периодических и со временем периодических точек данного уравнения. Тогда любая точка $x_0 \in I \setminus Y$ апериодическая с вероятностью единица.*

Список литературы

[1] Свирежев Ю.М., Логофет Д.О. Устойчивость биологических сообществ. М.: Наука, 1978. 352 с.

[2] Шарковский А.Н., Коляда С.Ф., Сивак А.Г., Федоренко В.В. Динамика одномерных отображений. Киев: Наукова думка, 1989. 216 с.

[3] Родина Л.И. Об отталкивающих циклах и хаотических решениях разностных уравнений со случайными параметрами // Труды Института математики и механики УрО РАН. 2016. Т. 22, № 2. С. 227–235.

Топологический подход к мониторингу переходных состояний в кардиодинамике

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На настоящий момент электрокардиографическое обследование является наиболее распространённым методом оценки кардиодинамики. Несмотря на достижения в анализе биомедицинских сигналов, в электрокардиографической диагностике остаётся ряд нерешённых проблем. Одна из них состоит в том, что визуальная оценка ЭКГ позволяет установить наличие лишь отдельных клинически интерпретируемых особенностей работы сердца. Остаётся круг ситуаций, в которых имеющиеся методы анализа ЭКГ не дают результатов, поддающихся корректной интерпретации. К таким ситуациям, в частности, относятся случаи так называемой «переходной кардиодинамики», когда диагностически значимые изменения на ЭКГ выражены нечётко.

Нами был разработан топологический подход к анализу вейвлет-спектров ЭКГ сигнала, позволяющий классифицировать многообразие форм ЭКГ с топологических позиций [патент RU2632756C2]. Разработанный топологический подход позволяет детектировать переходные состояния кардиодинамики, имеющие место в момент смены топологического типа вейвлет-спектра ЭКГ. В качестве примера в докладе будет рассмотрен вариант топологической классификации, подразумевающей подразделение ЭКГ сигналов на 20 топологических типов. Использование данного подхода демонстрируется на ряде примеров, отвечающих различной кардиодинамике. В частности, будут рассмотрены сценарии начального развития инфаркта миокарда и сценарии возникновения мерцательной аритмии. С помощью разработанного подхода найдены индикативные показатели, имеющие определённое диагностическое значение. Разработан и запущен в эксплуатацию интернет-сервис, позволяющий в онлайн режиме анализировать ЭКГ-сигналы (in-silico.ru/services/).

Использование топологических методов позволяет расширить возможности по выявлению и предупреждению критических состояний в кардиодинамике. Развитый подход может представлять интерес не только в связи с анализом ЭКГ сигналов, но и при анализе любых сигналов, имеющих периодическую структуру.

Классификация расширений квазипериодических потоков на торе

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Доклад посвящен вопросам топологической классификации потоков на трехмерном торе, порождаемых векторными полями вида

$$\dot{\varphi} = \omega, \qquad \dot{\theta} = v(\varphi, \theta), \tag{1}$$

где (φ, θ) – угловые координаты на торе $\mathbf{T}^3$, вектор $\omega \in \mathbf{R}^2$ имеет рационально независимые компоненты, $v(\varphi, \theta)$ – непрерывная, периодическая по всем переменным функция.

Будем рассматривать задачу классификации для потоков с одним и тем же вектором ω. Класс таких потоков обозначим через $\mathcal{C}_\omega$. Классификация потоков из этого класса производится по отношению *послойной топологической сопряженности*, т.е. потоки f^t и g^t сопряжены, если существует гомеоморфизм тора $h : \mathbf{T}^3 \to \mathbf{T}^3$ такой, что $h(\varphi, \theta) = (\varphi, H_\varphi(\theta))$, где $H_\varphi : S^1 \to S^1$ – гомеоморфизм S^1, и $h(f^t(\varphi, \theta)) = g^t(\varphi, H_\varphi(\theta))$. Если же отображение H_φ не гомеоморфизм, а только гомоморфизм (непрерывное отображение на), то говорят о *послойной полусопряженности* потоков f^t и g^t. Пусть $\Phi(t, \varphi) = \varphi + \omega t$, $\Theta(t, \varphi, \theta) = \theta + F^t(\varphi, \theta)$, – поднятие на универсальное накрытие тора $\mathbf{T}^3$ потока f^t. Известно, что любой поток из класса $\mathcal{C}_\omega$ имеет единственный *вектор вращения* $\rho = (\omega_1, \omega_2, \varrho)$, $\quad \varrho = \lim_{t\to\infty} F^t(\varphi, \theta)/t$, где ϱ (*число вращения слоя*) не зависит от начальных данных (φ, θ) (см. [1],[2]). Другой важной характеристикой, используемой при классификации, является свойство *регулярности*: поток $f^t \in \Phi_\omega$ регулярен, если существует число $c > 0$ такое, что $|F^t(\varphi, \theta) - t\varrho| < c$. Вектор вращения ρ называется *нерезонансным*, если его компоненты рационально независимы. В противном случае он называется *резонансным*.

В докладе формулируется ряд уточнений, касающихся классификации потоков из $\mathcal{C}_\omega$. Будем говорить, что поток $f^t \in G_\omega$ *послойно дистален*, если для всех $\varphi \in \mathbf{T}^2$ и $\theta_1 \neq \theta_2$ существует $\delta(\theta_1, \theta_2) > 0$ такое, что $\lim_{t\to\pm\infty} |\Theta(t, \varphi, \theta_1) - \Theta(t, \varphi, \theta_2)| \geq \delta(\theta_1, \theta_2)$. Если же в слоях существуют асимптотические траектории, то поток называется *послойно проксимальным*.

Теорема. *Пусть поток $f^t \in \mathcal{C}_\omega$ регулярен и его вектор вращения нерезонансный. Тогда*

1. *если поток f^t послойно дистален, то он топологически сопряжен линейному $(\varphi, \theta) \mapsto (\varphi + \omega, \theta + \varrho)$;*

2. *если поток f^t послойно проксимален, то он топологически полусопряжен линейному потоку.*

Доказательство этого результата опирается на обобщенную теорему Готшалка-Хедлунда [5] о существовании непрерывных решений аддитивных гомологических уравнений.

Список литературы

[1] *Веременюк В.В.* Существование числа вращения уравнения $\dot{x} = \lambda(t, x)$ с периодической по x и почти периодической по t правой частью // Дифференц. уравнения. 1991. Т. 27, № 6, С. 1073–1076,

[2] *Веременюк В.В.* Формула для числа вращения уравнения первого порядка с квазипериодической по времени правой части // Дифференц. уравнения. 1995. Т. 31, № 1. С. 158–159.

[3] *Jäger T., Stark J.* Towards a classification for quasiperiodically forced circle homeomorphisms // J. London Math. Soc. 2006. V. 73, n. 2. P. 727–744.

[4] *Jäger T.* Linearization of conservative toral homeomorphisms // Inventiones mathematicae. 2009. V. 176, n. 3. P. 601–616.

[5] *Aliste-Prieto J., Jäger T.* Almost periodic structures and the semiconjugacy problem // J. Differential Equations. 2012. V. 252. P. 4988–5001.

Об одном классе движений осесимметричного спутника в гравитационом поле

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Сообщение посвящено свойствам движения осесимметричного спутника относительно центра масс под действием гравитационного момента. Центр масс спутника движется по круговой орбите в центральном гравитационном поле.

Пусть **L** - вектор кинетического момента спутника относительно его центра масс. Если проекция **L** на ось симметрии спутника равна нулю, возможны «плоские» движения - движения, в которых ось симметрии лежит в плоскости орбиты, а вектор угловой скорости ω перпендикулярен этой плоскости.

В фазовом пространстве гамильтоновой системы с двумя степенями свободы, описывающей движение осесимметричного спутника относительно центра масс, плоским движениям отвечают фазовые траектории, лежащие на двумерном инвариантном многообразии. Поведение фазовых траектрий на этом многообразии аналогично поведению траекторий на фазовом портрете математического маятника - сепаратрисы разделяют траектории, соответствующие вращениям и колебаниям спутника относительно местной вертикали.

Плоские движения осесимметричного спутника изучались в [1]. В нашем анализе некоторые новые свойства плоских движений и движений, к ним близких, были установлены на основе общего подхода к исследованию гамильтоновых систем с сепаратрисными контурами, развиваемого в [2,3].

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Список литературы

[1] Маркеев А.П.: Устойчивость плоских колебаний и вращений спутника на круговой орбите. *Космич. исслед.*, **13**, 322-336 (1975)

[2] Lerman, L.M.: Hamiltonian systems with loops of a separatrix of a saddle-center. *Sel. Math. Sov.*, **10**, 297-306 (1991)

[3] Grotta Ragazzo, C.: On the stability of double homoclinic loops. *Commun. Math. Phys.*, **184**, 251-272 (1997)